

GENERAL POSITION ARGUMENT AND PICTURE-VALUED INVARIANTS

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We shall present a general construction which allows one to get invariants of various topological and geometric objects valued in groups G_n^k and their relatives.

Groups G_n^k were introduced by the author in [1]. For two integers $n > k$, we define the group G_n^k as the group having the following $\binom{n}{k}$ generators a_m , where m runs the set of all unordered k -tuples m_1, \dots, m_k , whereas each m_i are pairwise distinct numbers from $\{1, \dots, n\}$ modulo the following three types of relations:

For each $(k + 1)$ -tuple U of indices $u_1, \dots, u_{k+1} \in \{1, \dots, n\}$, consider the $k + 1$ sets $m^j = U \setminus \{u_j\}$, $j = 1, \dots, k + 1$. With U , we associate the relation

$$a_{m^1} \cdot a_{m^2} \cdots a_{m^{k+1}} = a_{m^{k+1}} \cdots a_{m^2} \cdot a_{m^1}; \quad (1)$$

for two tuples U and \bar{U} , which differ by order reversal, we get the same relation.

For k -tuples m, m' with $Card(m \cap m') < k - 1$, consider the *far commutativity relation*:

$$a_m a_{m'} = a_{m'} a_m \quad (2).$$

Note that the far commutativity relation can occur only if $n > k + 1$.

Besides that, for all multiindices m , we write down the following relation:

$$a_m^2 = 1 \quad (3)$$

These groups naturally appear when we have a *general position dynamical system* consisting of n particles, with respect to some general conditions such that during the motion of points, there are finitely many moments when some k particles are not in general position. Such configurations naturally lead to multiindices m of the generators. If we consider a *general position deformation* (homotopy) between two dynamical systems, we have finitely many moments when more complicated singularities occur. These exactly lead to “codimension 2” conditions and the relations of the group.

Dynamical systems with fixed number k of points lead to invariants valued in groups G_n^k and can be studied by (virtual) knot theory methods.

Groups G_n^k share many nice properties with braid groups and have various homomorphisms to free groups [2], which leads to powerful and easy-to-calculate invariants of dynamical systems. A crucial example first mentioned in [1] and considered in detail in [2] is the dynamical system of points on the plane where $k = 3$ and the generators of G_n^3 correspond to triples of points on the same line, which corresponds to horizontal trisecants of the corresponding braid.

If we allow the number of particles to vary in such a way that at some moments, two particles can get born or get cancelled, then this leads to more complicated objects which can be studied by (virtual) *tangles and links*.

Thus, *dynamics with fixed number of points* can be studied by using *braids* and *dynamics when the number of points changed* can be studied by *tangles and links*.

The “link” (knot) counterpart of G_n^2 -braids is known as *free knots*. Free knots possess various invariants valued in pictures (knot diagrams) [3] and free groups, [1].

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Such dynamical systems are general enough. This approach has various applications in topology, geometry, combinatorial group theory, differential equations:

invariants of virtual braids and knots and their relatives play the role of invariants of such objects.

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