ON TUNNEL NUMBERS OF COMPOSITE KNOTS

RUIFENG QIU

Let k be a knot in the 3–sphere S^3 and let $E(k) = S^3 - N(k)$, where $N(k)$ is an open tubular neighborhood of k. Then there is always a collection of disjoint and embedded arcs $\tau_1, \ldots, \tau_t$ in S^3, such that $\tau_i \cap k = \partial \tau_i$ for each i and $H = S^3 - N((\cup_{i=1}^t \tau_i) \cup k)$ is a handlebody. This means that ∂H is a Heegaard surface of $E(k)$. These arcs $\tau_1, \ldots, \tau_t$ are called unknotting tunnels of k and we say that they form a tunnel system for k. The tunnel number of k, denoted by $t(k)$, is the minimal number of arcs in a tunnel system for k. Let $g(E(k))$ be the Heegaard genus of the knot exterior $E(k)$. Clearly $g(E(k)) = t(k) + 1$.

Let k_1 and k_2 be two knots in a 3-sphere. We denote by $k_1 \sharp k_2$ the connected sum of k_1 and k_2. The super additivity question of tunnel number is the following:

$$t(k_1 \sharp k_2) = t(k_1) + t(k_2) + 1?$$

This talk can be taken as a short survey on the super additivity question of tunnel number.

References

[1] Xutao Gao; Qilong Guo and Ruifeng Qiu, A note on tunnel number of composite knots. *Topology Appl.* Vol. 158, 2240-2243, 2012.

[2] Tsuyoshi Kobayashi and Yo'av Rieck, Knot exteriors with additive Heegaard genus and Morimoto's conjecture. *Algebr. Geom. Topol.* Vol. 8, 953-969, 2008.

[3] Kanji Morimoto, There are knots whose tunnel numbers go down under connected sum. *Proc. Amer. Math. Soc.* Vol. 123, 3527-3532, 1995.

[4] Kanji Morimoto, On the super additivity of tunnel number of knots. *Math. Ann.* Vol. 317, 489-508, 2000.

[5] Tao Li and Ruifeng Qiu, On the degeneration of tunnel numbers under connected sum. *To appear in Trans. Amer. Math. Soc.*

Department of Mathematics, East China Normal University, Dongchuan Road No. 500 Shanghai, 200241, China

E-mail address: rfqiu@math.ecnu.edu.cn