ON STABILITY OF A CERTAIN GEOMETRIC CRITERION OF QUASICONFORMALITY IN THE PLANE

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Given a comlex number $\lambda \neq 0, 1$ consider the local homeomorphic mapping $f : D \to f(D) \subset \bar{\mathbf{C}}$ of a domain $D \subset \bar{\mathbf{C}}$. A mapping f is said to have the property $\mathfrak{R}[\lambda; \delta]$ $(\delta \geq 0)$ provided that any point $z \in D$ has an open neighborhood $U \subset D$ such that $f|U$ is injective and for every quadruple $\{z_1, z_2, z_3, z_4\}$ of distinct points in U with anharmonic ratio $[z_1 : z_2 : z_3 : z_4] = \lambda$ the estimate

$$|[f(z_1) : f(z_2) : f(z_3) : f(z_4)] - \lambda| \leq \delta \cdot \min\{|\lambda|, |1 - \lambda|\}$$

holds. The case $\delta = 0$ was investigated in papers [1]-[2], where the Möbius property has been derived from the condition $\mathfrak{R}[\lambda; 0]$ for local injective and Borel measurable mappings. In particular, $\mathfrak{R}[\lambda; 0]$ implies 1-quasiconformality of f. The following theorem gives the stability of that geometric criterion.

Theorem. Given $\lambda \in \mathbf{C} \setminus \{0, 1\}$ and $\psi(\lambda) := 16|\lambda|^2 + 88|\lambda| + 96$ the property $\mathfrak{R}[\lambda; \delta]$ with $0 \leq \delta < 1/\psi(\lambda)$ implies the local K-quasiconformality of f with the following upper estimate

$$K[f] \leq \sqrt{1 + \delta^2(\psi(\lambda))^2} + \delta \cdot \psi(\lambda)$$

for it's coefficient of quasiconformality. In case $\lambda = a + ib \neq 0, 1$ with $b \neq 0$ the following estimate

$$K[f] \leq 1 + \delta\sqrt{1 + \frac{\min\{a^2, (1-a)^2\}}{b^2}}$$

is also true. In particular, $K[f] \leq 1 + \delta$ when $a \in \{0, 1\}$ and $b \neq 0$.

References

[1] V. Aseev, T. Kergylova, "On transformations that preserve fixed anharmonic ratio", *Tokyo J. Math.*, 33, No. 2, pp. 365-371 (2010).

[2] T. A. Kergilova, "Injective Borel-measurable mappings preserving a prescribed cross-ratio up to the complex conjugation are necessarily Möbius transformations" (Russian. English summary), *Vestn. Novosib. Gos. Univ., Ser. Mat. Mekh. Inform.* 10, No. 4, 68-81 (2010).

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