

# COMPLEXITY OF HYPERBOLIC 3-MANIFOLDS

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The most useful approach to a classification of 3-manifolds is the complexity theory founded by S. Matveev [1]. Unfortunately, exact values of complexity are known for few infinite series of 3-manifold only.

We present the results on complexity for two infinite series of hyperbolic 3-manifolds with boundary. The first is a family of Paoluzzi – Zimmermann manifolds from [1], and the second is its analogy defined in [4]. These manifolds are constructing as follows.

For every  $n \geq 3$  consider an  $n$ -gonal bipyramid  $\mathcal{B}_n$ , the union of pyramids  $NL_0L_1 \dots L_{n-1}$  and  $SL_0L_1 \dots L_{n-1}$  along the common  $n$ -gonal base  $L_0L_1 \dots L_{n-1}$ . Let  $k$  be such integer that  $0 \leq k < n$ . The first family corresponds to the case  $\gcd(n, 2 - k) = 1$ , and the second – to the case  $\gcd(n, 2 - k) = 2$ . Let us identify the faces of  $\mathcal{B}_n$  in pairs: for each  $i = 0, \dots, n - 1$  the face  $L_iL_{i+1}N$  gets identified with the face  $SL_{i+k}L_{i+k+1}$  by a homeomorphism of faces. (indices are taken mod  $n$  and the vertices are glued together in the order in which they are written). Denote the resulting identification spaces by  $M_{n,k}^*$ . It is an orientable pseudomanifold with one singular point. Cutting of a cone neighborhood of the singular point from  $M_{n,k}^*$  we get a compact manifold  $M_{n,k}$  with one boundary component.

Denote by  $c(M_{n,k})$  the Matveev’s complexity of  $M_{n,k}$  which is defined as the minimum possible number of true vertices of an almost simple spine of  $M_{n,k}$ .

**Theorem.** [3, 4] Suppose that  $\gcd(n, 2 - k) = 1$  or  $\gcd(n, 2 - k) = 2$ . Then for every integer  $n \geq 6$  we have  $c(M_{n,k}) = n$ .

## REFERENCES

- [1] S. Matveev, *Algorithmic topology and classification of 3-manifolds*, Springer, (2007).
- [2] L. Paoluzzi, B. Zimmermann, “On a class of hyperbolic 3-manifolds and groups with one defining relation”, *Geom. Dedicata*, 60, 113-123 (1996).
- [3] A. Vesnin, E. Fominykh, “Exact values of complexity for Paoluzzi - Zimmermann manifolds”, *Doklady Mathematics*, 84, No. 1, 542-544 (2011).
- [4] A. Vesnin, E. Fominykh, “On complexity of hyperbolic 3-manifolds with geodesic boundary”, *Siberian Math. Journal*, 53, No. 4, 625-634 (2012).

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