MINIMAL SURFACES ON CARNOT GROUPS

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The introduction of suitable notions of variation, graph and surface measure is one of main problems in research of non-holonomic minimal surfaces. We solve them for Lipschitz (in sub-Riemannian sense) mappings $\varphi : \mathbf{G} \to \ell$ defined on measurable subsets of Carnot groups with values on integral line ℓ of an arbitrary vector field X_i of a degree l. We give a definition of a graph mapping φ_Γ that assigns to each point x the element $\exp\Bigl(\varphi(x)X_i\Bigr)(x)$, generalize the concept of sub-Riemannian differentiability introduced in [1] and obtain the notion of polynomial hc-differentiability, and prove that graph mappings are polynomially hc-differentiable at almost every point of D. These results are applied to the proof of the area formula for the "interior" measure $\mathcal{H}^\nu_\Gamma$ of a graph:

$$\int\limits_D \sqrt{1+|\widehat{D}\varphi(y)|_i^2}\, d\mathcal{H}^\nu(y) = \int\limits_{\varphi_\Gamma(D)} d\mathcal{H}^\nu_\Gamma(x).$$

Moreover, we find analytic description of some non-holonomic minimal surfaces. For mappings $\varphi : D \to \ell$ defined on a domain $D \subset \mathbf{G}$ that satisfies some additional requirements, we define a class

$$\mathcal{F}_{K,\varphi} = \Bigl\{\xi = \varphi + \psi : \int\limits_D \frac{|\widehat{D}\psi(x)|_i^2}{\Bigl(1+|\widehat{D}\varphi(x)|_i^2\Bigr)^{\frac{3}{2}}}\, d\mathcal{H}^\nu(x) \geq K\|\psi\|_4^2\Bigr\}, \quad K>0,$$

where second horizontal derivatives of φ are measurable functions. It defines class of graphs $\mathcal{S}_{K,\varphi}$ of mappings from $\mathcal{F}_{K,\varphi}$. Then minimal surfaces in the class $\mathcal{S}_{K,\varphi}$ are surfaces of zero sub-Riemannian mean curvature defined by functions φ, that is,

$$\sum_{j=1}^{\dim V_1} X_j \Bigl\langle \frac{X_j\varphi(x)}{\sqrt{1+|\widehat{D}\varphi(x)|^2}} \Bigr\rangle = 0$$

for almost all $x \in D$. This case is new and it differs essentially from that studied in [2]. See [3] and [4] for details.

Список литературы

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The research was supported by the State Maintenance Program for Young Russian Scientists and the Leading Scientific Schools of Russian Federation (Grant NSh-921.2012.1), and FCPK (application № 2012-1.2.2-12-000-1001-022).