VISUALIZATION OF THE 8 HOMOGENEOUS 3-GEOMETRIES AND THE THURSTON CONJECTURE

EMIL MOLNÁR

It is well-known that the classical Euclidean and non-Euclidean 3-geometries of constant curvature: $\mathbf{E^3}$, $\mathbf{S^3}$ and $\mathbf{H^3}$ can be modeled in the real projective space $\mathbf{P^3}$ (or projective sphere $\mathbf{PS^3}$, respectively), i.e. in the subspace structure of a real vector space $\mathbf{V^4}$ and in its dual. That means, the usual projections (parallel or central ones) from $\mathbf{E^3}$ into the (moveable) computer screen $\mathbf{E^2}$ are also possible.

The author extended this method to the other 5 homogeneous 3-geometries

$$\mathbf{S^2} \times \mathbf{R},\ \mathbf{H^2} \times \mathbf{R},\ \sim \mathbf{SL(2,R)},\ \mathbf{Nil} \ \text{ and } \ \mathbf{Sol}$$

(the so-called Thurston geometries) as well. Thus, visualization of these (strange) spaces, animations in them are possible, due to my colleagues István PRÓK, Jenä SZIRMAI [2],[3],[4] and our students (e.g. János PALLAGI and Benedek SCHULTZ [6]).

Interesting pictures to the famous Thurston conjecture or to other problems, by visualizations, can help us in the present and future investigations. Some of them will also be illustrated in this talk: E.g. one parameter tilings in $\mathbf{E^3}$ and in the Bolyai-Lobachevskian hyperbolic space $\mathbf{H^3}$ (I. Prok and J. Szirmai [3]). The densest lattice-like geodesic ball packing in **Nil** space (whose density 0,78 is larger than the corresponding Euclidean one 0,74, J. Szirmai and his students [6],[7]). Some (geodesic and translation) balls in $\sim$**SL(2,R)** and in **Sol** geometry will also be presented by international collaboration (Blaženka DIVJAK, Zlatko ERJAVEĆ, Barnabás SZABOLCS, Brigitta SZILÁGYI [1]).

Our collaboration with the colleagues in Novosibirsk [5] promises further attractive results and pictures as well.

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BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS, BUDAPEST, H–1521, HUNGARY
E-mail address: emolnar@math.bme.hu