

SUPPORT FUNCTIONS OF THE CONVEX POLYHEDRON IN LOBACHEVSKY'S SPACE

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The convex geometry of the Euclidean space plays an important role in mathematical analysis. The convex geometry of the Lobachevsky space H_κ^n with the curvature $(-\kappa)$, where $\kappa > 0$, is a natural expansion of the convex geometry of the Euclidean space. There is a natural correspondence between closed convex subsets $Q \subset H_\kappa$ and conformally flat metrics $ds^2 = \frac{dx^2}{h_Q^2(x)}$, $x \in R^{n-1}$, which are defined on $\overline{R^{n-1}}$ and have bounded one-dimensional curvature:

$$-\frac{\kappa}{2} \leq h_Q \frac{d^2 h_Q}{d\xi^2} - \frac{1}{2} |\nabla h_Q|^2 \leq \frac{\kappa}{2}, \quad (1)$$

where ∇f is the gradient of f in R^{n-1} , $\frac{d^2 f}{d\xi^2}$ is the second derivative of f in R^{n-1} along a unit vector $\xi \in R^{n-1}$. These metrics are called support functions of Q in this paper. The following formula is true for a finite convex polyhedron of the Lobachevsky space:

$$h_Q(x) = \min_i \{h_{\Delta_i}(x)\} \quad (2)$$

where $h_{\Delta_i}(x)$ are support functions of $(n-1)$ -dimensional sides of the border of Q . The calculation of the functions $h_{\Delta_i}(x)$ occurs recurrently and is reduced to a case when Δ_i are k -dimensional simplexes in H_κ^n ($k < n$). Such functions we will call elementary conformally flat splines. Usual splines-functions of many variables are limited to the functions of cellular structure [1].

These functions are known as the functions which range of definition is divided into cells (in a flat case - rectangles, triangles, etc., in multidimensional - parallelepipeds, pyramids, etc.). In each cell a function is defined somewhat in the homogeneous way with conditions of smoothness along borders of cells. In difference from usual spline-functions the representation (2) of the function $h_Q(x)$ by conformally flat spline-functions has other nature, here it is not required to specify the range of the definition of $h_{\Delta_i}(x)$. Function $h_Q(x)$ has smoothness $C^{1,1}$ and any function $f \in C^1$ can be approached as much as precisely by a function $h_Q(x)$ of a type (2) in the norm of the space C^1 on a compact subset (at big enough κ). The theory of conformal-flat splines is based on the connection between conformally flat metrics of the bounded curvature and convex subsets in Lobachevsky's space. The conformally flat splines correspond convex polyhedrons in Lobachevsky's space thus [2,3]. The conformally flat spline also has "a cellular structure" which is defined in the parameters $\{\Delta_i\}$, entering into the formula for the function $h_Q(x)$ and corresponding to cellular structure of the border of a convex polyhedron of the Lobachevsky's space.

The obvious formula (2) for function $h_Q(x)$ allows to simplify a calculation and make it more effective: it is not necessary to break the range of definition of a function and it is possible to use parallel algorithms for the calculation of elementary splines $h_{\Delta_i}(x)$. Conformally flat spline-functions are most effective at the decision of problems of mathematical physics in which the conformally flat metrics are present naturally (for example, at problems of the tomography, geophysics, acoustics, integrated geometry).

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The program complex in the environment of MatLab, and also independently program complex on C++ Builder for the calculation of representation (2) of functions of many variables by the conformally flat spline-functions are constructed in this paper.

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