Homomorphic images of Coxeter groups in the Golod 2-group

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The Coxeter group is a group, generated by a finite number of involutions, the set of defining relations which consists only of the degree of the product every two generators.

In 1959, P. S. Novikov announced the existence of finitely generated infinite groups limited exponenta. While preparing the proof of this result, E. S. Golod built for every prime p and every $n \geq 2$ infinite n-generated p-group, [1]. Recall that the basis for the construction of this group is a sufficient condition infinite dimensionality of the quotient algebra of the free associative algebra $F^{(1)}$ of polynomials without constant term of n non-commuting variables over an arbitrary field by a homogeneous ideal I. This condition limits the number of each degree polynomials in the generating set of the ideal I. Each polynomial of algebra $F^{(1)}$ is being built in sufficiently large so that homogeneous components of this degree are taken as generators of I and the number of each of these generators degree satisfies the above conditions is infinite-dimensional. So being built nonnilpotent nilalgebra $A = F^{(1)}/I$. Algebra A over a field of characteristic p under the operation $\circ$:

$$a \circ b = a + b + ab, \quad a, b \in A,$$

forms an infinite p-group, which is called the adjoint group of algebra A. We denote by $x_1, x_2, \ldots, x_n$ free generators of $F^{(1)}$, and let $a_j = x_j + I$, $j = 1, 2, \ldots, n$. Then the subgroup $\langle a_1, a_2, \ldots, a_n \rangle$ of adjoint group of A will also be infinite. We call it the Golod group.

The Golod group is residually finite and orders its elements are not bounded. There are Golod groups with infinite center, [2], and the Golod group with trivial center, [3, 4]. As shown in [5], the ideal I can be constructed so, that each $(n-1)$-generated subgroup of Golod group is finite. On the other hand, for each $n = 2, 3, \ldots$ and each odd prime q is constructed n-generated Golod q-group with infinite subgroup, generated by two conjugate elements of prime order, [6, 7]. Constructed Golod 2-group with infinite subgroup generated by a pair of conjugate elements of the fourth order, [7]. These subgroups can be constructed so as to satisfy the sufficient condition for infinite index subgroup of Golod group, [8]. The same properties can have subgroups of the following theorem.

Theorem 1. *For each $n \geq 3$ and for each $m \geq 3$ the n-generated Golod 2-group with infinite subgroup, generated by m involutions, is constructed.*

References

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