DOMINIONS OF TORSION FREE ABELIAN GROUPS IN METABELIAN GROUPS

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The dominion of a subalgebra H in an universal algebra A (in a class $\mathcal{M}$) is the set of all elements $a \in A$ such that for all homomorphisms $f, g : A \to B \in \mathcal{M}$ if f, g coincide on H, then $a^f = a^g$.

We write

$$\mathrm{dom}_A^{\mathcal{M}}(H) = \{a \in A \mid \forall M \in \mathcal{M}, \forall f, g : A \to M,$$
$$\text{if } f\mid_H = g\mid_H \text{ then } a^f = a^g\}$$

Here, as usual, $f\mid_H$ denotes the restriction of f on H.

The concept of dominions was introduced by J. R. Isbell (1965) to study epimorphisms. Later dominions were investigated in several classes of algebras (G. M. Bergman 1990; D. Wasserman 2001; H. E. Scheiblich 1976; B. Mitchell 1972; D. Saracino 1983; A. Magidin 1999 — (9 papers); S. Shakhova 2005, 2006, 2010). There is a connection between dominions and amalgams. See the survey article by P. M. Higgins (1988) for the details.

It is not hard to see that $\mathrm{dom}_A^{\mathcal{M}}(-)$ is a closure operator on the lattice of subalgebras of A, in the sense that it is extensive (the dominion of H contains H), indempotent (the dominion of the dominion of H equals the dominion of H), and isotone (if $H \subset K$, then the dominion of H is contained in that of K).

The notion of a closed subgroup arose.

We say that a group H is closed (or absolutely closed) in class $\mathcal{M}$ of groups if for every $G \in \mathcal{M}$ for each embedding $H \leq G$ we have

$$\mathrm{dom}_G^{\mathcal{M}}(H) = H.$$

In this paper we study closed groups in the variety $\mathcal{A}^2$ of metabelian groups.

Theorem 1. Every nontrivial torsion free abelian group is not closed in the class of metabelian groups.

Theorem 2. Let a group H be a torsion free group, G be a metabelian group and $H \leq G$. If $H \cap G' = (1)$ then $\mathrm{dom}_G^{\mathcal{A}^2}(H) = H$.

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