ON DIAMETERS OF THE CAYLEY GRAPHS OF THE SYMMETRIC GROUPS

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Let S_n be the symmetric group of degree n and $x_1 = (1, 2)$, $x_2 = (2, 3)$, $\ldots$, $x_{n-1} = (n-1, n)$, $x_n = (1, n)$ be generators of S_n and $X = \{x_1, x_2, \ldots, x_n\}$.
We proved the following

Theorem. *The diameter of the Cayley graph of S_n relative to X is equal*

(i) $\dfrac{n^2}{4}$ *for even $n = 2k$ and corresponds to the only one permutation*

$$\begin{pmatrix} 1 & 2 & \ldots & k-1 & k & k+1 & k+2 & \ldots & 2k-1 & 2k \\ k+1 & k+2 & \ldots & 2k-1 & 2k & 1 & 2 & \ldots & k-1 & k \end{pmatrix};$$

(ii) $\dfrac{n^2-1}{4}$ *for odd $n = 2k+1$ and corresponds to the only two permutations*

$$\begin{pmatrix} 1 & 2 & \ldots & k & k+1 & k+2 & k+3 & \ldots & 2k & 2k+1 \\ k+1 & k+2 & \ldots & 2k & 2k+1 & 1 & 2 & \ldots & k-1 & k \end{pmatrix},$$

$$\begin{pmatrix} 1 & 2 & \ldots & k-1 & k & k+1 & k+2 & \ldots & 2k & 2k+1 \\ k+2 & k+3 & \ldots & 2k & 2k+1 & 1 & 2 & \ldots & k & k+1 \end{pmatrix}.$$

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