Some criteria for supersolubility of finite groups

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All groups considered in this paper are finite. Let H be a subgroup of a group G. The *permutizer* [1, p. 27] of H in G is the subgroup $P_G(H) = \langle x \in G \mid \langle x\rangle H = H\langle x\rangle\rangle$.

Definition. Let H be a subgroup of a group G. We say that

1) H is *permuteral* in G, if $P_G(H) = G$;

2) H is *strongly permuteral* in G, if $P_U(H) = U$ whenever $H \leq U \leq G$.

There exists groups which have permuteral but not strongly permuteral subgroups. For example, in the group $G = PSL(2, 7)$ a Sylow 3-subgroup Z_3 is permuteral in G. Since $Z_3 \leq U \leq G$, where U is isomorphic to the alternating group A_4 of degree 4 and $P_U(Z_3) = Z_3$, then Z_3 is not strongly permuteral in G.

Theorem 1. *Let G be a metanilpotent group. Then the following statements are equivalevt*:

1) *G is supersoluble*;

2) *Every Sylow subgroup of G is strongly permuteral in G*;

3) *Every Sylow subgroup of G is permuteral in G.*

Theorem 2. *Let G be a group. Then the following statements are equivalevt*:

1) *G is supersoluble*;

2) *Every pronormal subgroup of G is strongly permuteral in G*;

3) *Every pronormal subgroup of G is permuteral in G*;

4) *Every Hall subgroup of G is strongly permuteral in G*;

5) *Every Hall subgroup of G is permuteral in G.*

Theorem 3. *Let G be a group. Then the following statements are equivalevt*:

1) *G is supersoluble*;

2) *$G = AB$ is the product of strongly permuteral nilpotent subgroups A and B of G*;

3) *$G = AB$ is the product of permuteral nilpotent subgroups A and B of G.*

Corollary 3.1. *Let G be a group, and let $G = AB$ be a product of its Sylow subgroups A and B. Then G is supersolvable if and only if A and B is permuteral in G.*

Corollary 3.2. *Let G be a group. Then G is supersoluble if and only if $G = F(G)H$, where H is a permuteral Carter subgroup of G.*

For details, see [2].

References

1. Between nilpotent and solvable / M. Weinstein (Editor) Passaic: Polygonal Publ. House, 1982.

2. A.F. Vasil'ev, V.A. Vasil'ev, T.I.Vasil'eva, On permutizers of subgroups of finite groups, arXiv:1305.2630v1 [math.GR] 12 May 2013.