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ON DEGREES OF IRREDUCIBLE LINEAR GROUPS

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Let G be a finite group, A is a group of its automorphisms such that $(|G|, |A|) = 1$. Then A is called *a group of cosimple automorphisms* of the group G and the semidirect product $\Gamma = GA$ of the groups G and A is a group. If $C_G(a) = C_G(A)$ for each element $a \in A^{\#}$, then A is said to be *a strong-centralized group of cosimple automorphisms* of the group G.

Condition B. Let us say that the group $\Gamma = GA$ satisfies Condition B, if $G \triangleleft \Gamma$, $(|A|, |G|) = 1$, A is an odd-order group that is not normal in the group Γ, $C_G(a) = C_G(A) = C$ for each element $a \in A^{\#}$, and the group G has faithful irreducible complex character of degree n, which is a-invariant for at least one element $a \in A^{\#}$.

From the theorem, proved in the series of papers [1]-[3], it is obvious that if $n < 2|A|$ and A is of odd-order, then $n = |A| - 1$, $|A| + 1$, $2(|A| - 1)$ or $2|A| - 1$ and n is a degree of a certain prime number. The paper [4] states if the group Γ satisfies Condition B and $n = 2|A| + 1$, then n is also a prime power. Hence, n is divisible by the degree f of a certain prime number such that $f \equiv -1$ or $1(\mathrm{mod}|A|)$. [3] hypothesizes the fairness of this statement for an arbitrary number n.

Suppose if $|A| = p$ is a prime number, then from the above-mentioned theorem, we obtain Isaacs's [5] result for the groups having the above-named property and the appropriate result obtained by Newton [6] follows from the hypothesis.

Theorem. *Assume the group Γ satisfies Condition B. Then n will divide by the degree f of a certain prime number such that $f \equiv -1$ or $1 (mod|A|)$.*

[1] Yadchenko A. A. *On Π-solvable irreducible linear groups with a Hall $TI-$subgroup of odd order.* Part I // Works of the Institute of Mathematics of NASB. 2008. V. 16, No. 2. P. 118-130.

[2] Yadchenko A. A. *On Π-solvable irreducible linear groups with a Hall $TI-$subgroup of odd order.* Part II // Works of the Institute of Mathematics of NASB. 2009. V. 17, No. 2. P. 94-104.

[3] Yadchenko A. A. *On Π-solvable irreducible linear groups with a Hall $TI-$subgroup of odd order.* Part III // Works of the Institute of Mathematics of NASB. 2010. V. 18, No. 2. P. 99-114.

[4] A. A. Yadchenko. *On Irreducible Linear Groups of Nonprimary Degree* // ISRN Algebra. V. 2011. Article ID 868096. 20 pages doi: 10.5402/2011/868096.

[5] Isaacs I. M. *Complex p-solvable linear groups* // J. Algebra. 1973. V. 24. N 3. P. 513-530.

[6] Benjamin Newton. *On the degrees of complex p-solvable linear groups* // Journal of Algebra 288 (2005) 384-391.