Unipotent elements from subsystem subgroups of type A_2 in representations of the special linear group

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Let K be an algebraically closed field of odd characteristic p, G be a simply connected algebraic group of type A_r over K, $r \geq 3$, let ω_i, $1 \leq i \leq r$, be the fundamental weights with respect to a fixed maximal torus $T \subset G$ and a choice of a Borel subgroup $B \supset T$.

A rational irreducible representation of G with the highest weight $\sum_{i=1}^{r} a_i\omega_i$ is called p-restricted if and only if all $a_i < p$. A subsystem subgroup of a semisimple algebraic group is a subgroup generated by root subgroups associated with all roots from a certain subsystem of its root system.

Suppose that $H \subset G$ is a subsystem subgroup of type A_2, $x \in H$ is a regular unipotent element, φ is a rational irreducible p-restricted representation of G, $\omega = \sum_{k=1}^{r} a_k\omega_k$ is the highest weight of φ, ω^* is the highest weight of the representation dual to φ, $\mathrm{J}_\varphi(x)$ is the set of Jordan block sizes for $\varphi(x)$ (without their multiplicities), $s(\varphi) = \min_{1 \leq i \leq r-1}(2a_i + 2a_{i+1})$, $m(\varphi) = \min(\sum_{k=1}^{r} 2a_k + 1, p)$, $\mathbb{N}$ is the set of nonnegative integers.

Theorem 1 *Let G, x, and φ be the same as above and $s(\varphi) < p$. Then*

$$\{k \in \mathbb{N} \mid 1 \leq k \leq m(\varphi), k \equiv m(\varphi) (\mathrm{mod}\ 2)\} \subset \mathrm{J}_\varphi(x),$$

except the following cases:

(i) $r = 3$, $\omega = a_2\omega_2$ *and* a_2 *is odd;*

(ii) $r = 3$, ω *or* $\omega^* = a_1\omega_1 + a_2\omega_2 + a_3\omega_3$, $2a_1 + 2a_2 < p$, $a_2 + a_3 = p - 1$, *and* a_1 *is odd;*

(iii) $p = 3$ *and* $s(\varphi) = 2$.

In such cases

$$\{k \in \mathbb{N} \mid 3 \leq k \leq m(\varphi), k \equiv m(\varphi) (\mathrm{mod}\ 2)\} \subset \mathrm{J}_\varphi(x)$$

and in the case (i)

$$1 \notin \mathrm{J}_\varphi(x).$$

This result is a part of the programme of investigating the behavior of unipotent elements in modular representations of classical algebraic groups.

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