AN ANALOGUE OF THE FRATTINI ARGUMENT FOR HALL SUBGROUPS

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The following simple statement is frequently used in the finite group theory.

The Frattini Argument. *Let A be a normal subgroup of a finite group G and let S be a Sylow p-subgroup of A for a prime p. Then $G = AN_G(S)$.*

Let π be a set of primes. A subgroup H of a group G is called a *π-Hall subgroup* if every prime divisor of $|H|$ belongs to π and $|G : H|$ is not divisible by the elements in π.

According to [1], we say that a group G *satisfies* $\mathscr{E}_\pi$ if there exists a π-Hall subgroup in G.

It is easy to show that if A is a normal subgroup of a finite group G and H is a π-Hall subgroup of G then $H \cap A$ is a π-Hall subgroup of A.

The following statement is the main result of the talk:

Theorem *Let π be a set of primes, A be a normal subgroup of a finite $\mathscr{E}_\pi$-group G. Then A possesses a π-Hall subgroup H such that $G = AN_G(H)$.*

We also provide examples showing that the condition $G \in \mathscr{E}_\pi$ in Theorem is essential and that the equality $G = AN_G(H)$ need not hold for every π-Hall subgroup H of A.

References

[1] P. Hall, Theorems like Sylow‘s, Proc. London Math. Soc., 6:22 (1956), 286–304.

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