Automorphisms of divisible rigid groups

Denis Ovchinnikov

A group G is said to be rigid if it has a normal series $G = G_1 > G_2 > \ldots > G_n > G_{n+1} = 1$ in which each factor G_i/G_{i+1} is an abelian group and torsion free as $\mathbb{Z}[G/G_i]$- module. Rigid groups are introduced in [1] in connection with algebraic geometry over groups. Important examples of rigid groups are free solvable groups. G is called divisible (see [2]) if all G_i/G_{i+1} are divisible modules. Any $\mathbb{Z}[G/G_i]$ is a (right) Ore domain and we denote by $Q(G_i/G_{i+1})$ the (right) Ore skew field of fractions of $\mathbb{Z}[G/G_i]$. Therefore for a divisible rigid group G the quotient G_i/G_{i+1} can be viewed as a vector space over $Q(G_i/G_{i+1})$. G is called splittable if it splits into a semidirect product $A_1A_2\ldots A_n$ of abelian groups $A_i \cong G_i/G_{i+1}$. A splittable divisible rigid group is uniquely defined by dimensions of A_i over $Q_i = Q(A_1 \ldots A_{i-1})$ (denoted by α_i). It is proved that every divisible rigid group is splittable [3] and any rigid group is embedable into some divisible rigid group [2].

We study the groups of automorphisms of divisible rigid groups.

Theorem 1 *For a divisible rigid group G the $Aut(G) \cong MN$ is a semidirect product of $M = GL_{\alpha_1}(Q_1) \cdot \ldots \cdot GL_{\alpha_n}(Q_n)$ and a normal subgroup $N = \{\phi_g \mid g \in A_2 \ldots A_n\} \cong A_2 \ldots A_n$, which is a group of corresponding inner automorphisms.*

The second result concerns with normal automorphisms of G. Recall that $\phi \in Aut(G)$ is normal if for any normal subgroup H of G, $\phi(H) = H$.

Theorem 2 *The group of normal automorphisms of G is equal to $Inn(G) \times \mathbb{Z}_2$ where $\mathbb{Z}_2 = \langle e_0 \rangle$ and $e_0 : a_i \to a_i$ for $i < n$, $e_0 : a_n \to a_n^{-1}$, $a_i \in A_i$.*

References

[1] A.Myasnikov, N. Romanovskiy, Krull dimension of solvable groups, J.Algebra, 324, N 10, 2010, pp. 2814-2831.

[2] N. S. Romanovskiy, Divisible rigid groups. (Russian) Algebra Logika 47, N 6, 2008, pp. 762-776; translation in Algebra Logic 47, N 6, 2008, pp. 426-434.

[3] A.Myasnikov, N. Romanovskiy, Logical aspects of divisible rigid groups, to appear.