# On finite $\pi$-solvable group with supersolvable $\pi$-Hall subgroup

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All groups considered in this paper will be finite. Each $\pi$-solvable group $G$ has a subnormal series $G = G_0 \supseteq G_1 \supseteq \ldots \supseteq G_{n-1} \supseteq G_n = 1$, whose factors $G_{i-1}/G_i$ are $\pi'$-groups or abelian (or nilpotent) $\pi$-group. The least number of abelian (nilpotent) $\pi$-factors of all such subnormal series of a group $G$ is called the derived (respectively nilpotent) $\pi$-length of a $\pi$-solvable group $G$ and is denote by $l^a_\pi(G)$ (respectively by $l^n_\pi(G)$). Clearly, $l^n_\pi(G) \leq l^a_\pi(G)$ for any $\pi$-solvable group $G$. Some estimates of these $\pi$-lengths are established in [1]–[4]. In particular, if $G$ is a $\pi$-solvable group in which the derived subgroup of a $\pi$-Hall subgroup is nilpotent, then $l^n_\pi(G) \leq 1 + \max_{r\in\pi} l_r(G)$, see [1]. We received an analogue this results for the derived $\pi$-length.

**Theorem 1.** *Let $G$ be a $\pi$-solvable group. If the derived subgroup of $\pi$-Hall subgroup of $G$ is nilpotent, then $l^a_\pi(G) \leq 1 + \max_{r\in\pi} l^a_r(G)$.*

Since the derived subgroup of a supersolvable group is nilpotent, then from Theorem 1 follows

**Corollary 1.** *Let $G$ be a $\pi$-solvable group. If a $\pi$-Hall subgroup of $G$ is supersolvable, then $l^a_\pi(G) \leq 1 + \max_{r\in\pi} l^a_r(G)$.*

**Corollary 2.** *Let $G$ be a $\pi$-solvable group. If a Sylow $p$-subgroup of $G$ is cyclic for every $p \in \pi$, then $l^a_\pi(G) \leq 2$.*

**Corollary 3.** *Let $G$ be a $\pi$-solvable group, and let a Sylow $p$-subgroup of $G$ be bicyclic for every $p \in \pi$. If $2 \notin \pi$, then $l^a_\pi(G) \leq 3$.*

# References

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