Some linear methods in the study of almost fixed-point-free automorphisms

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There are very strong results on finite groups G with a fixed-point-free automorphism φ: for example, if $C_G(\varphi) = 1$ and φ has prime order p, then G is nilpotent by Thompson's theorem, and the nilpotency class is bounded in terms of p by the Higman–Kreknin–Kostrikin theorem. If a finite group G admits an automorphism $\varphi \in \operatorname{Aut} G$ with nontrivial fixed-point subgroup $C_G(\varphi)$ which is 'small' in some sense (so that φ is 'almost fixed-point-free'), then it is natural to expect that the group G 'almost' enjoys similar nice properties, which usually means that there is a subgroup of bounded index that is soluble, or nilpotent, or has bounded Fitting height, or bounded nilpotency class, etc. For finite groups, the classification often reduces such studies to the case of soluble G. Then representation theory is used to obtain bounds for the Fitting height of 'almost entire' G. For nilpotent groups, Lie ring methods are used for bounding the nilpotency class of G or of a suitable subgroup. There remain several important open problems, some even in the case $C_G(\varphi) = 1$. The first part of the talk is a survey of results and open problems in this area. The second part focuses on applying 'semisimple' results on fixed-point-free automorphisms of Lie rings to 'unipotent' situations (when a p-group acts on a p-group). In particular, we present most recent results on Frobenius groups of automorphisms with 'unipotent' kernels.