On the embedding problem for generalized Baumslag-Solitar groups

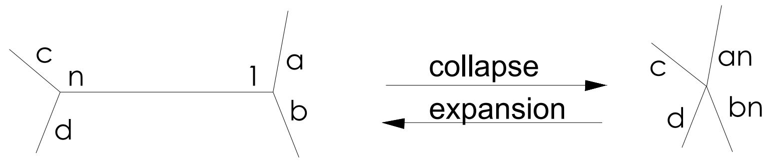
Finitely generated group G is called *generalized Baumslag–Solitar group (GBS group)* if G is acting on a tree T with infinite cyclic edge and vertex stabilizers. Then, by Bass–Serr theorem, group G is a $\pi_1(\mathbb{A})$ – fundamental group of some graph of groups $\mathbb{A}$ (see, for example, [1]), with vertex and edge groups is infinite cyclic.

A GBS group can be described by a labeled graph $\mathbb{A} = (\Gamma, \lambda)$, there Γ is a finite graph and $\lambda\colon E(\Gamma) \to \mathbb{Z} \setminus \{0\}$ is a labeling of edges of Γ. Label $\lambda(e)$ written on edge e with origin v determine embedding $\alpha_e\colon e \to v^{\lambda(e)}$ of cyclic edge group $\langle e \rangle$ into cyclic vertex group $\langle v \rangle$.

GBS groups have been studied from the different postions [2], [3], [4]. In particular, the isomorphism problem for GBS groups was discussed: to define algorithmically when two given labeled graphs set isomorphic GBS groups. In spite of the fact that in some private cases the problem of isomorphism was solved [5], [6], [7], generally the existence of algorithm wasn't established.

We study *embedding problem* for GBS groups: to define algorithmically when two labeled graphs $\mathbb{A}_1 = (\Gamma_1, \lambda_1)$ and $\mathbb{A}_2 = (\Gamma_2, \lambda_2)$ define such GBS groups that group $\pi_1(\mathbb{A}_1)$ is embedable into $\pi_1(\mathbb{A}_2)$.

There are two basic deformations of labeled graph that doesn't change it's fundamental groups – expansions and collapses:



A labeled graph $\mathbb{A}$ is reduced if it does not admit a collapse move. An elementary deformation is a finite sequence of collapse and expansion moves. Given a labeled graph $\mathbb{A}$, the deformation space $\mathcal{D}_{\mathbb{A}}$ of $\mathbb{A}$ is the set of all labeled graphs related to $\mathbb{A}$ by an elementary deformation. We prove

Theorem. Let $\mathbb{A}_1, \mathbb{A}_2$ be labeled graphs. If there is only finitely many reduced labeled graphs in $\mathcal{D}_{\mathbb{A}_1}$ then embedding problem $\pi_1(\mathbb{A}_1) \to \pi_1(\mathbb{A}_2)$ is decidable.

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