On the existence of arc-transitive distance regular covers of cliques with $\lambda = \mu$ related to Suzuki groups

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Let Γ be a distance regular graph of diameter 3 and let μ denote the number of common neighbours for any two vertices of Γ at distance 2. Recall that if graph Γ is antipodal then Γ is a r-fold cover of $(k+1)$-clique, Γ has intersection array $\{k, \mu(r-1), 1; 1, \mu, k\}$, and any two adjacent vertices of Γ have exactly $\lambda = k - 1 - \mu(r-1)$ common neighbours, where parameter k is the degree of graph Γ [2].

A graph is called arc-transitive (or edge-symmetric) if its automorphism group acts transitively on ordered pairs of adjacent vertices.

Arc-transitive antipodal distance regular graphs of diameter 3 with $\lambda = \mu$ were described in [1]. In particular in [1] there were found three new potential series of graphs corresponding to groups $Sz(q), U_3(q), {}^2G_2(q)$.

In the present work we prove the existence of infinite series of distance regular graphs related to groups $Sz(q)$, where $q = 2^{2a+1} > 2$, which appears to be new.

Suppose that a non-normal subgroup H of a group G and an element $g \in G - H$ are given. Let $\Gamma(G, H, HgH)$ denote the graph with vertex set $\{Hx \mid x \in G\}$ whose edges are the pairs $\{Hx, Hy\}$ such that $xy^{-1} \in HgH$.

The following theorem holds.

Theorem 1. *Let $G = Sz(q)$, where $q = 2^{2a+1} > 2$, let $S \in Syl_2(G)$ and let g be an involution of G not contained in S. Then $\Gamma(G, S, SgS)$ is arc-transitive antipodal distance regular graph with intersection array $\{q^2, q^2 - q - 2, 1; 1, q + 1, q^2\}$.*

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References

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