On finite simple nonabelian groups of Lie type over fields of different characteristics with the same prime graph

Zinov'eva M. R.

Let G be a finite group, $\pi(G)$ be the set of all prime divisors of its order, $\omega(G)$ be the *spectrum* of G, i.e. the set of its element orders. The set $\omega(G)$ defines a graph with the following relation of adjacency: different vertices r and s from $\pi(G)$ is joined by an edge if and only if $rs \in \omega(G)$. This graph is called the *Gruenberg-Kegel graph* or the *prime graph* of G and denotes by $GK(G)$.

In [1] A.V. Vasil'ev posed the problem 16.26:

Does there exist a positive integer k such that there no k pairwise nonisomorphic finite nonabelian simple groups with the same graph of primes? Conjecture: $k = 5$.

It is easy to see that there exist four pairwise nonisomorphic finite nonabelian simple groups with the same prime graph, namely: J_2, A_9, $C_3(2)$, $D_4(2)$.

In [2] it is investigated the case when the alternating group A_n for $n \geq 5$ and a finite simple group have the same prime graph.

In the present abstract we consider two finite simple groups of Lie type over fields of different characteristics p_1 и p_2. Using information about prime graphs of finite simple groups from [3]–[6], and orders of groups of Lie type, which can be found, for example, in [7], we obtain the following result.

Proposition 1. *Let $G_i = A_{n_i-1}(q_i)$, $n_i \geq 7$, n_i is odd, q_i is a power of an odd prime p_i for $i \in \{1, 2\}$, $p_2 \neq p_1$ and $GK(G_1) = GK(G_2)$. Then $n_1 = n_2$ and the following condition holds:*

either p_i is a primitive prime divisor of ${q_j}^3 - 1$, or $n_{p_i} = (q_j - 1)_{p_i}$ and p_i is a primitive prime divisor of $q_j - 1$ for $\{i, j\} = \{1, 2\}$.

Proposition 2. *Let $G_1 = A_{n_1-1}(q_1)$, $n_1 \geq 7$, n_1 is odd, $G_2 = B_{n_2-1}(q_2)$, $n_2 \equiv 0, 1 \pmod 4$, $n_1 \geq 8$, q_i is a power of an odd prime p_i for $i \in \{1, 2\}$, p_i are primes, $p_2 \neq p_1$ and $GK(G_1) = GK(G_2)$. Then $n_1 = 3n_2/2 + 1$ or $n_1 = 3n_2/2 + 3/2$ and the following conditions hold:*

(1) $n_1 \equiv 4, 5, 9 \pmod{12}$;

(2) *p is a primitive prime divisor of $q_1^6 - 1$;*

(3) *p_1 is a primitive prime divisor of $q^j - 1$, $j \in \{1, 2, 3\}$.*

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Список литературы

[1] Nonsolved problems of group theory. Kourovka notebook. 16-th ed. Novosibirsk: Novos. st. univ., 2006.

[2] **Zvezdina M.A.** On nonabelian simple groups having the same prime graph as an alternating group // Siberian Math. J. 2013. V. 54, № 1. C. 47–55.

[3] **Kondrat'ev A.S.** On prime graph components for finite simple groups // Math. Sb. 1989. V. 180, № 6. P. 787–797.

[4] **Williams J. S.** Prime graph components of finite groups // J. Algebra. 1981. Vol. 69, no. 2. P. 487–513.

[5] **Vasil'ev A.V., Vdovin E.P.** An adjacency criterion for two vertices of the prime graph of a finite simple group // Algebra and Logic. 2005. V. 44, № 6. P. 381–406.

[6] **Vasil'ev A.V., Vdovin E.P.** Cocliques of maximal size in the prime graph of a finite simple group // Algebra and Logic. 2011. V. 50, № 4. P. 291–322.

[7] **Kondrat'ev A.S.** Groups and Lie algebras. Ekaterinburg: UB RAS, 2009. 310 p.