ON THE NONABELIAN COMPOSITION FACTORS OF A FINITE GROUP THAT IS PRIME SPECTRUM MINIMAL

N. V. MASLOVA, D. O. REVIN

We consider finite groups only.

Let G be a finite group. The set $\pi(G)$ of all prime divisors of the number $|G|$ is called the *prime spectrum* of G. A finite group G is *prime spectrum minimal* if $\pi(H) \neq \pi(G)$ for every proper subgroup H of G.

A subgroup H of a group G is called a *Hall subgroup* if $(|H|, |G : H|) = 1$. The class of finite groups in which all maximal subgroups are Hall is a proper subclass of the class of finite prime spectrum minimal groups.

P. Shumyatsky has written down to "Kourovka notebook" [1] (problem 17.125) the following

Conjecture 1. In any finite group G, there is a pair a, b of it's conjugate elements such that $\pi(G) = \pi(\langle a, b\rangle)$.

It is easy to prove, the Shumyatsky conjecture is equivalent to the following

Conjecture 2. Every prime spectrum minimal group is generated by a pair of conjugate elements.

In [2] we have proved, any finite group with Hall maximal subgroups is generated by a pair of conjugate elements. Thus we have obtained the partially solution of Problem 17.125 from "Kourovka notebook". In our proof we used the description of nonabelian composition factors of a finite group with Hall maximal subgroups [3]. Thus it is interesting the following

Problem. What are nonabelian composition factors of finite groups that are prime spectrum minimal?

The main result of this work is the following

Theorem. *Let $S = A_n$ where $n \geq 5$. Then the following conditions hold*:
(i) *S is not isomorphic to a composition factor of a prime spectrum minimal group if n is not a prime*;
(ii) *S is a prime spectrum minimal group if n is a prime.*

Following [4] let us define $c(G)$ to be the least integer n such that there exist proper subgroups $H_1, \dots, H_n$ such that $\pi(G) = \pi(H_1) \cup \ldots \cup \pi(H_n)$.

In [4] it was proofed, $c(A_n) \leq 2$ for each n.

Using main theorem of this work we obtain

Corollary. *Let $S = A_n$ where $n \geq 5$. Then the following conditions hold*:
(i) $c(S) = 1$ *if n is not a prime*;
(ii) $c(S) = 2$ *if n is a prime.*

Acknowledgement. The research was supported by The work is supported by RFBR (projects 13-01-00469 and 13-01-00505), by and the Program of the Joint Investigations of the Ural Division of the Russian Academy of Sciences with Siberian Division of the Russian Academy of Sciences (project 12--1-10018) and with Belorussian National Academy of Sciences (project 12-C-1-1009), by the grant of the President of Russian Federation for young scientists (project MK-3395.2012.1) and by a grant from the IMM of UD RAS for young scientists in 2013.

References

[1] The Kourovka notebook. Unsolved problems in group theory. Edited by V. D. Mazurov and E. I. Khukhro. 17-th. ed., Russian Academy of Sciences Siberian Division, Institute of Mathematics, Novosibirsk, 2010.

[2] *Maslova N. V., Revin D. O.* Generation of a finite group with Hall maximal subgroups by a pair of conjugate elements // Trudy Inst. Mat. i Mekh. UrO RAN, 2013. V. 19, no. 3, to appear (in Russian).

[3] *Maslova N. V.* Nonabelian composition factors of a finite group whose all maximal subgroups are Hall. Siberian Math. J. 2012. V. 53, no. 5. P. 853-861.

[4] *Belonogov V. A.* On sets of subgroups of a finite simple group, which control its the prime spectrum // Trudy Inst. Mat. i Mekh. UrO RAN, 2013. V. 19, no. 3, to appear (in Russian).

N. V. Maslova, IMM UB RAS, Ekaterinburg
E-mail address: butterson@mail.ru

D. O. Revin, IM SD RAS, Novosibirsk
E-mail address: danila.revin@gmail.com