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On a semigroup of π-normal Fitting classes

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All groups considered are finite. In definitions and notation we follow [1].

A normally hereditary class of groups $\mathfrak{F}$ is called a Fitting class if it is closed under the products of normal $\mathfrak{F}$-subgroups. If $\emptyset \neq \mathfrak{F}$ is a Fitting class then a subgroup $G_{\mathfrak{F}}$ of a group G is called its $\mathfrak{F}$-radical if it is the largest normal $\mathfrak{F}$-subgroup of G. A non-empty Fitting class $\mathfrak{F}$ is called normal in a Fitting class $\mathfrak{X}$ or $\mathfrak{X}$-normal (this is denoted by $\mathfrak{F} \trianglelefteq \mathfrak{X}$) if $\mathfrak{F} \subseteq \mathfrak{X}$ and for every $\mathfrak{X}$-group G a subgroup $G_{\mathfrak{F}}$ is $\mathfrak{F}$-maximal in G. Recall that the product of Fitting classes $\mathfrak{X}$ and $\mathfrak{Y}$ is the class of groups $\mathfrak{X}\mathfrak{Y} = (G : \ G/G_{\mathfrak{X}} \in \mathfrak{Y})$ which is also a Fitting class.

In the class $\mathfrak{S}$ of all soluble groups it is established [2] that the product of Fitting classes $\mathfrak{F}$ and $\mathfrak{H}$ is $\mathfrak{S}$-normal when at least one multiplier is normal in $\mathfrak{S}$. Since the operation of multiplication of Fitting classes is associative, this fact signifies that the algebra $< \mathcal{P}, \cdot >$ is a semigroup ($\mathcal{P}$ is the set of all $\mathfrak{S}$-normal Fitting classes and "$\cdot$" is the operation of multiplication of Fitting classes). Later the fact similar to the mentioned above was proved [3] for the case of $\mathfrak{E}$-normal Fitting classes where $\mathfrak{E}$ denotes the class of all groups. Note that for the case of $\mathfrak{S}_\pi$-normal Fitting classes ($\mathfrak{S}_\pi$ is the class of all soluble π-groups) the result [2] was extended in [4]. This leads us to the question whether such property holds in going from soluble groups to arbitrary ones.

Let $\mathbb{P}$ be the set of all primes, $\emptyset \neq \pi \subseteq \mathbb{P}$ and let $\mathfrak{E}_\pi$ be the class of all π-groups. If a Fitting class $\mathfrak{F}$ is normal in $\mathfrak{E}_\pi$ then we call it π-normal.

The following theorem gives a positive answer to the question whether the algebra $< \mathcal{P}, \cdot >$ is a semigroup, where $\mathcal{P}$ is the set of all π-normal Fitting classes.

Theorem. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be Fitting classes such that $\mathfrak{X} \subseteq \mathfrak{E}_\pi$ and $\mathfrak{Y} \trianglelefteq \mathfrak{E}_\pi$. Then $\mathfrak{X}\mathfrak{Y} \trianglelefteq \mathfrak{E}_\pi$.*

In the case $\pi = \mathbb{P}$ the theorem implies the result of Laue (see theorem 2.7 [3]).

References

[1] Doerk K., Hawkes T. Finite soluble groups. Berlin–New York : Walter de Gruyter. — 1992. — 891 p.

[2] Cossey J. Products of Fitting Classes // Math. Z. — 1975. — Bd. 141. — S. 289.–295.

[3] Laue H. Über nichtaflösbare normale Fittingklassen // J. Algebra. — 1977. — 45. — P. 274.–283.

[4] Savelyeva N.V. Maximal subclasses of π-normal Fitting classes // Herald of Polotsk State University (Series C. Fundamental Sciences). — 2008. — No. 9. — P. 22.–31 (in Russian).