Some properties of linear groups

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One of the results in extremal combinatorics which has been proved and extended in many ways is the Erdös-Ko-Rado theorem [3]. A k-set system is a collection of subsets of $\{1, 2, ..., n\}$ of size k. A k-set system is intersecting if its elements are pairwise non-disjoint.

Theorem 1. (Erdös, Ko and Rado [3]) *Let k and n be positive integers such that $n \geq 2k$.*

(1) *If $\mathfrak{F}$ is an intersecting k-set system on an n-set, then $|\mathfrak{F}| \leq \binom{n-1}{k-1}$.*

(2) *If $n > 2k$, then $\mathfrak{F}$ meets this bound if and only if $\mathfrak{F}$ is the collection of all k-subsets containing a fixed element $i \in \{1, ..., n\}$.*

Erdös-Ko-Rado theorem has been extended in many ways. In [6], Hsieh investigated the analogous problem for finite vector spaces. Let $GF(q)$ denote a finite field with q elements.

Theorem 2. (Hsieh [6]) *Let $\mathfrak{F}$ be a family of k-dimensional subspaces of an n-dimensional vector space V over $GF(q)$ such that the members of $\mathfrak{F}$ intersect pairwise non-trivially.*

(1) *If $n \geq 2k$, then $|\mathfrak{F}| \leq \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}_q$, where the Gaussian coefficient $\begin{bmatrix} n-1 \\ k-1 \end{bmatrix}_q$ denotes the number of k-dimensional subspaces of V containing a specific 1-dimensional subspace of V.*

(2) *If $n > 2k$, then $\mathfrak{F}$ meets this bound if and only if $\mathfrak{F}$ is a family of k-dimensional subspaces of V containing a specific 1-dimensional subspace of V (see* [6, Theorem 4.4]*).*

Fix $t \in \mathbb{N}$. In 1986, Frankl and Wilson [4] generalized Hsieh's results for the family $\mathfrak{F}$ of k-dimensional subspaces of V such that for any $A, B \in \mathfrak{F}$, $\dim(A \cap B) \geq t$.

Let Ω be a finite set and G a permutation group on it. A subset A of G is intersecting if for any $\delta, \tau \in A$, there exists $x \in \Omega$ such that $\delta(x) = \tau(x)$. As an intersecting set of the permutation group G, we can name the stabilizer of a point. The Erdös-Ko-Rado theorem for permutation groups is finding the size of the largest intersecting set of G. This problem goes back to 1977 [2].

Theorem 3. *Let $\mathfrak{F}$ be an intersecting set of the symmetric group $\mathbb{S}_n$.*

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(1) (Deza and Frank [2]) $|\mathfrak{F}| \leq (n-1)!$.

(2) ([1, 5, 7, 9]) $\mathfrak{F}$ meets this bound if and only if $\mathfrak{F}$ is a coset of the stabilizer of a point.

In [8], it has been proved that the size of intersection set of the permutation group $PGL_2(q)$ acting on the projective line $\mathbb{P}_q$, is at most $q(q-1)$ and the only sets S that meet this bound are the cosets of the stabilizer of a point of $\mathbb{P}_q$. Also, Guo and Wang (An Erdöos-Ko-Rado theorem in general linear groups, arXiv:1107.3178) find the upper bound for the size of the intersecting set of $GL_n(q)$ acting on $(GF(q))^n - \{0\}$. In the submitted paper, we study the Erdös-Ko-Rado theorem for $SL_2(q)$ and $GL_2(q)$ acting on $(GF(q))^2 - \{0\}$ and $PSL_2(q)$ acing on the projective line $\mathbb{P}_q$. In this talk we concern the Erdös-Ko-Rado theorem for some linear groups and symplectic groups.

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