Logical Aspects of the Theory of Rigid Solvable Groups (abstract)

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A group G is said to be m-rigid if it has a normal series

$$G = G_1 > G_2 > \ldots > G_m > G_{m+1} = 1$$

with abelian factors each of which G_i/G_{i+1}, viewed as an $\mathbb{Z}[G/G_i]$-module, has no torsion. Free solvable groups are rigid. A rigid group G is called divisible if any factor G_i/G_{i+1} is a divisible module over the ring $\mathbb{Z}[G/G_i]$ or, in other words, it is a vector space over skew field of fractions of this ring.

We say for m-rigid groups that H is embedded into G with preserving linear independence, if any system elements of H_i/H_{i+1} linear independent over the ring $\mathbb{Z}[H/H_i]$ has to be linear independent over the ring $\mathbb{Z}[G/G_i]$.

Theorem 1 *Arbitrary m-rigid group can be embedded with preserving linear independence into some divisible m-rigid group.*

Malcev proved that a free solvable group of length ≥ 2 has undecidable elementary theory. The universal theory of a free metabelian group is decidable (Chapuis).

Theorem 2. *The universal theory of a free solvable group of length ≥ 4 is undecidable.*

For the class Σ_m of rigid groups of length $\leq m$ we define algebraic closed objects: G is called algebraic closed if for any embedding $G \hookrightarrow H$ in this class with preserving linear independence any system of equations over $x_1, \ldots, x_n$ with coefficients from G has a solution in G^n if and only if it has a solution in H^n. G is called existential closed if for any such embedding any $\exists$-formula is true on G if and only if it is true on H.

Theorem 3. *Algebraic closed groups in Σ_m are exactly divisible m-rigid groups, they are also existential closed objects in Σ_m.*

We study elementary theories of divisible m-rigid groups and construct a system of axioms in group theory signature which defines exactly all divisible m-rigid groups. Denote by T corresponding theory.

Fix some countable divisible m-rigid group M. We note that this group is constructible. Extend the signature of group theory by constants from M. We add some recursive system of axioms which means that M is embedded into given rigid group with preserving linear independence. Denote corresponding theory by T_M.

Theorem 4. *The theories T and T_M are complete and therefore decidable.*

Theorems 3 and 4 were proved joint with Alexei Myasnikov.