On the Heritability of the Property D_π by Subgroups in case $2 \in \pi$

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Let π иу a set of primes. A subgroup H of a finite group G is called a π-Hall subgroup if every prime divisor of $|H|$ belongs to π and $|G : H|$ is not divisible by elements in π. According to [1], we say that a finite group G satisfies D_π (or G is a D_π-group), if maximal π-subgroups of G are all conjugate. Notice that Sylow theorem implies that maximal π-subgroups in D_π-groups are π-Hall subgroups.

We consider the following problem 17.44(b) from "The Kourovka notebook" [2].

Problem. Does an overgroup of a π-Hall subgroup in a D_π-group satisfy D_π?

Using the classification of finite simple groups we obtain an affirmative answer to the problem in case $2 \in \pi$.

Theorem. Let π be a set of primes and $2 \in \pi$. Suppose a finite group G satisfies D_π and H is a π-Hall subgroup of G. Then every subgroup M of G with $H \leq M$ satisfies D_π.

1. P. Hall Theorems like Sylow's // Proc. London Math. Soc. Ser. 3. 1956. V. 6, no. 22. P. 286–304.

2. The Kourovka notebook. Unsolved problems in group theory // Edited by V. D. Mazurov and E. I. Khukhro. 17-th. ed. IM SD RAS, Novosibirsk. 2010.