A derived π-length of π-soluble groups and central π-Hall intersections

O.A. Shpyrko

Only finite groups are considered. Let G be a π-soluble group. The smallest natural number of abelian π-factors among all subnormal series of group G with π'-factors or abelian π-factors is called a derived π-length of G and is defined by $l_\pi^a(G)$. It's clear that $l_\pi(G) \leq l_\pi^n(G) \leq l_\pi^a(G)$ for all π-soluble group G and in case $\pi = \{p\}$ we have $l_\pi^a(G) = l_\pi^n(G) = l_\pi(G) = l_p(G)$.

Recall, under the central intersection of π-Hall subgroups means the intersection of two different π-Hall subgroups containing the center of one of them.

In works [1-3] provided estimates of the p-length $l_p(G)$ of p-soluble group G (π-length $l_\pi(G)$ and nilpotent π-length $l_\pi^n(G)$ of π-soluble group G) depending on the structure of central p-Sylow (π-Hall) intersections.

Similar results were obtained for the derived π-length of π-soluble groups.

Theorem. 1. *If the π-soluble group G has no central π-Hall intersection, then $l_\pi^a(G) = d(G_\pi)$.*

2. *If in a π-soluble group G central π-Hall intersections are abelian either Schmidt group, then $l_\pi^a(G) \leq 1 + d(G_\pi)$.*

Corollary 1. *If the p-soluble group G has no central p-Sylow intersection, then $l_p^a(G) = d(G_p)$.*

Corollary 2. *Let G be a π-soluble group with metabelian central π-Hall intersections, then $l_\pi^a(G) \leq 3$.*

Corollary 3. *Let G be a π-soluble group with biabelian central π-Hall intersections, then $l_\pi^a(G) \leq 3$.*

References

1. Anizhenko A.G., Monakhov V.S., Central intersections and p-length of p-soluble groups// Docl.AS BSSR. 1977. T.XXI, №11. P.968-971.

2. Putilov S.V., About normal structure of finite π-soluble groups // Voprosy algebry. Minsk: University. 1990, V.5. P.82-86.

3. Shpyrko O.A., About intersections Hall subgroups and π-length of π-soluble group // VSU Vestnik. 2000. №4(18). P.82-89.