ON ISOMORPHISM BETWEEN TWO CONSTRUCTIONS OF ANTIPODAL DISTANCE-REGULAR GRAPHS

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For the background and necessary definitions, we refer the reader to [2]. In [1], two new constructions of antipodal distance-regular graphs have been proposed. The author of [1] remained the question whether these graphs were isomorphic to some known ones unsolved. Let us recall the main results from [1].

Let Γ_B be a graph with vertex set $B = g^G \cup (g^{-1})^G$, where g^G is a conjugacy class of elements of order p of group $G = PSL_2(p^n)$, and edge set $\{\{x, y\}|xy^{-1} \in B\}$, where p is an odd prime number, $q = p^n \geq 5$.

Theorem 1 (Mukhametyanov) *If $q \equiv 1(4)$, then the graph $\widehat{\Gamma_B}$ is is distance-regular with intersection array $\{q, q-3, 1; 1, 2, q\}$.*

Let Γ_J be a graph whose vertex set is the set of all elements of order p of group G and edge set $\{\{x, y\}|xy^{-1} \in J\}$, where J is a class of conjugate involutions of G.

Theorem 2 (Mukhametyanov) *If $q \equiv 1, 3(8)$, then the graph Γ_J is disconnected and its connected components are two isomorphic distance-regular graphs with intersection array $\{q, q-3, 1; 1, 2, q\}$.*

The following theorem is Proposition 12.5.3 from [2].

Theorem 3 (Mathon) *Let $q = rm + 1$ be a prime power, where $r > 1$ and either m is even or q is a power of 2. Let V be a vector space of dimension 2 over $F = F_q$ provided with a nondegenerate symplectic form B. Let K be the subgroup of the multiplicative group F^* of index r, and let $b \in F^*$. Then the graph $M(m, q)$ with vertex set $\{Kv \mid v \in V \setminus \{\overline{0}\}\}$ where $Ku \sim Kv$ if and only if $B(u, v) \in bK$ is distance-regular of diameter 3 with $r(q+1)$ vertices and intersection array $\{q, q-m-1, 1; 1, m, q\}$.*

In this work we show the following result.

Theorem 4 *The graph $\widehat{\Gamma_B}$ and each of connected components of Γ_J are isomorphic to the graph $M(2, q)$ with appropriate q.*

The work is supported by RFBR (project 12-01-31098), and by the grant of the President of Russian Federation for young scientists (project MK-1719.2013.1).

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