On subgroups which cover Frattini chief factors of finite group

Kamornikov S.F., Shemetkova O.L., Xiaolan Yi
Gomel Branch of International University MITSO, Belarus
Russian Economic University, Moscow, Russia
Zhejiang Sci-Tech University, Hangzhou, China

All groups considered in the paper are finite. We use standard definitions and notations from $[1-2]$.

Definition 1. An element x of a group G is called Q-superfrattini if it satisfies one of the following two equivalent conditions:

1) each chief factor A/B of G for which $x \in A \setminus B$ is Frattini;

2) each chief factor of G having a form $< x^G > /B$ is Frattini.

By definition, the identity element of G is Q-superfrattini.

Definition 2. A subgroup H of a group G is called a Φ-isolator if it covers all Frattini chief factors and avoids all non-Frattini chief factors of G.

A connection between Q-superfrattini elements of G and its Φ-isolators is investigated.

Theorem. For a group G, the following conditions hold:

1) any two Φ-isolators of G have the equal order;

2) if H is a Φ-isolator of G, then:

a) all elements of H are Q-superfrattini in G;

b) H is not contained properly in any subgroup of G whose elements are all Q-superfrattini in G;

c) $Core_G(H) = \Phi(G)$;

d) if $H \trianglelefteq G$, then $H = \Phi(G)$.

In 1962 Gaschütz in [3] introduced the concept of the prefrattini subgroup of a finite soluble group. In the original presentation the prefrattini subgroup is defined as the intersection of complements of the crowns of all non-Frattini chief factors of a fixed chief series of the group.

By definition, every soluble group has at least one prefrattini subgroup. As shown in [3], each prefrattini subgroup covers all Frattini and avoids all non-Frattini chief factors of a soluble group G, i.e. it is a Φ-isolator of G.

Proposition 1. Let H be a prefrattini subgroup of a soluble group G. Then all elements of H are Q-superfrattini in G.

Proposition 2. Let H be a Φ-isolator of a soluble group G. Then H is a prefrattini subgroup if and only if H permutes with every element of a Hall system of G.

References

1. Shemetkov L.A. Formations of Finite Groups. Moscow: Nauka, 1978.

2. Doerk K., Hawkes T.O. Finite Soluble Groups. Berlin - New-York: Walter de Gruyter, 1992.

3. Gaschütz W. Praefrattinigruppen // Arch. Math. 1962. Bd. 13, № 3. S. 418-426.