Cyclic unitary involutions on central simple algebras

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Let k be a field and K/k be a quadratic separable field extension. An involution on a central simple K-algebra A is called a unitary involution if its restriction to K is non-trivial and K/k-involution if this restriction is a non-trivial k-automorphism. The set of K/k-involutions of A will be denoted by $Inv_{K/k}(A)$.

Definition. *Let G be a finite cyclic group and $Inv_{K/k}(A) \neq \emptyset$. A central simple K-algebra A is called a cyclic involutorial crossed product with the group G if there exists a strictly maximal subfield N of A such that N is a composite of χ and K, where χ/k is a cyclic extension with the group G, χ and K are linear disjoint over k, and there exists $\tau \in Inv_{K/k}(A)$ such that $\tau|_\chi$ is trivial. In this case the corresponding involution τ is called cyclic.*

Cyclic involutions play an important role in investigating the structure of linear algebraic groups (especially, outer forms of groups of type A_n).

The problem of the existence of cyclic involutions on central simple algebras over global fields was solved affirmatively by Albert in [1]. He has also proved that for quaternion algebras all K/k-involutions are cyclic. Unfortunately, it is known that in general there are cyclic algebras such that all K/k-involutions are not cyclic. In [5] and [4] it is proved that all K/k-involutions on central simple K-algebras A are cyclic in the following cases:

(i) k is finite and the degree of A is odd;

(ii) k has a property that any quadratic k-form in 8 variables is isotropic, the degree of A is 3, and k contains a primitive 3th root of unity if $char k \neq 3$;

(iii) k is global and A is a split K-algebra of odd degree;

(iv) k is local non-dyadic and A is of odd degree.

The aim of the talk is to present the two following results.

Theorem 1. *Let k be a global field. Then all K/k-involutions on central simple K-algebras of odd degree are cyclic.*

By using results and methods from [2] and [3], we prove the following

Theorem 2. *Let k be the field of fractions of a two-dimensional excellent henselian local domain with an algebraically closed residue field of characteristic zero. Then all K/k-involutions on central simple K-algebras of odd degree are cyclic.*

References

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