# The Jordan block structure of unipotent elements in Weyl modules for groups of type $A_1$

T.S. Busel
Minsk, Institute of Mathematics,
National Academy of Sciences of Belarus
e-mail: tbusel@gmail.com

The Jordan block structure of unipotent elements in Weyl modules for an algebraic group of type $A_1$ is described. Let $K$ be an algebraically closed field of characteristic $p > 0$ and $G = A_1(K)$. The weight set of $G$ can be canonically identified with the set of integers. The following theorem is proved.

**Theorem 1.** *Let $M$ be the Weyl module for $G$ with highest weight $ip + j$ where $i$ is a nonnegative integer and $0 \leq j < p$. Then a nontrivial unipotent element has $i$ Jordan blocks of size $p$ and one block of size $j + 1$ on $M$.*

This result can be used for investigating the Jordan block structure of unipotent elements in modular representations of simple algebraic groups and finite Chevalley groups. Notice that an element of order $p$ in a simple algebraic group in characteristic $p$ almost always can be embedded into a Zariski closed subgroup of type $A_1$, only one conjugacy class in the group $G_2(K)$ for $p = 3$ yields an exception [1]. The construction of certain Weyl modules in the restrictions of irreducible modules for simple algebraic groups in characteristic $p$ to subgroups of type $A_1$ containing fixed elements of order $p$ was used in [2] for finding the minimal polynomials of these elements acting on such modules.

## References

[1] R. Proud, J. Saxl, and D. Testerman. Subgroups of type $A_1$ containing a fixed unipotent element in an algebraic group. — Journal of Algebra, 231 (2000), 53–66.

[2] I. D. Suprunenko, Minimal polynomials of elements of order $p$ in irreducible representations of Chevalley groups over fields of characteristic $p$. — Siberian Advances in Mathematics, 6 (1996), 97–150.