On the isomorphisms of crystallographic groups in pseudo-Euclidean space $\mathbb{R}^{3,3}$

V. Churkin

The crystallographic group is a subgroup G of motion group of pseudo-Euclidean space $\mathbb{R}^{p,q}$, such that the set Z of all translations in G is a lattice, i.e. is generated by translations on vectors of the fixed basis of space. The factor group G/Z is isomorphic to a subgroup of pseudo-orthogonal group $O_{p,q}(\mathbb{R})$.

In case of Euclidean spaces according to Bieberbach's theorem any isomorphism of crystallographic groups is affine, and automorphism is induced by linear transformation of space. In pseudo-Euclidean spaces it is not true, if $\min\{p,q\} \geqslant 3$ then there are crystallographic groups with non-standard nonlinear automorphisms [1]. These groups contain the abstract lattices: a subgroups which become lattices of translations at other realization of group as crystallographic group.

How many abstract lattices the crystallographic group can contain? This problem is solved at $p = q = 3$.

Theorem. *The crystallographic group G in pseudo-Euclidean space $\mathbb{R}^{3,3}$ can contain only one, two or three abstract lattices and no more than two non-standard automorphisms module subgroups of the linear automorphisms.*

The subgroup N of group G, generated by all abstract lattices, nilpotent of class one, two or three. If top and bottom series of N coincide then N define uniquely up to isomorphism.

The factor group G/N is respectively isomorphic to a subgroup of group $O_{p,q}(\mathbb{Z})$, $SL_3(\mathbb{Z})$ or $SL_2(\mathbb{Z})$ and can coincide with them.

References

1. V.A. Churkin, The weak Bieberbach theorem for crystallographic groups on pseudo-Euclidean spaces, *Siberian Mathematical Journal*, 51, N 3 (2010), 557 - 568.

Sobolev Institute of Mathematics of SB RAS, Novosibirsk,
Novosibirsk State University,
E-mail: churkin@math.nsc.ru