SPLITTING AUTOMORPHISMS OF FREE BURNSIDE GROUPS OF ORDERS p^k ARE INNER

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Automorphism φ of G is called *splitting automorphism of period* n, if $\varphi^n = 1$ and $g\, g^{\varphi} g^{\varphi^2} \cdots g^{\varphi^{n-1}} = 1$ for any element $g \in G$. BY theorem of O. Kegel [1] any finite group having a nontrivial splitting automorphism of prime period is nilpotent. E. Khukhro [2] proved that any solvable group having a nontrivial splitting automorphism of prime period also is nilpotent.

It is easy to verify that each inner automorphism of the periodic group of period n is its splitting automorphism of period n. However, the converse is not true.

In Kourovka Notebook S.V. Ivanov posed the question: *Let n is sufficiently large odd number and $m > 1$. Is it true that any splitting automorphism of period n of the free Burnside group $B(m, n)$ is inner* (see [3], question 11.36. b))?

In the paper [4] we proved that if the order of the automorphism φ of the free Burnside group $B(m, n)$ is prime, then φ is an inner automorphism. In [4] also was proved that if φ is an splitting automorphism of period n of $B(m, n)$, then the stabilizer of any normal subgroup $N \in \mathcal{M}_n$ under the action of $\langle\varphi\rangle$ is not trivial for any odd $n \geq 1003$, where $\mathcal{M}_n$ is a specially chosen set of normal subgroups. This statement has proved useful for solving mentioned problem for automorphisms of prime power orders. We proved the following result.

Theorem. *Let φ be splitting automorphism of period n of the free Burnside group $B(m, n)$, where $n \geq 1003$ is an arbitrary odd number. Then, if the order of the automorphism φ is prime power, then φ is an inner automorphism.*

References

[1] Kegel O. H., *Die Nilpotenz der H_p-Gruppen.* Math. Z.. 1961. Vol. 75 P. 373–376.

[2] Khukhro E. I., *Nilpotency of solvable groups admitting a splitting automorphism of prime order.* Algebra and Logic. 1980. V. 19:1 P. 77–84.

[3] *Kourovka Notebook; 11 ed.,* Novosibirsk, 1990.

[4] Atabekyan V. S., *Splitting automorphisms of free Burnside groups*, Mat. Sb., 204:2, 2013, P. 31–38.

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