One Generalization of Baer's Theorem

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All considered groups are finite. In [1] R. Baer studied properties of hypercenter $Z_\infty(G)$:

Theorem 1. *[1] For any group G the following conditions hold*

(1) If p is a prime then p-element $g \in Z_\infty(G)$ iff g permutes with all elements of G whose orders are coprime to the order of g.

(2) The hypercenter of G is the intersection of normalizers of all Sylow subgroups of G.

The following class is the natural generalization of nilpotent groups. Let π be a non-empty set of primes. Assume that $\{\pi_i | i \in I, i \neq j \Rightarrow \pi_i \cap \pi_j = \emptyset\}$ is a partition of π. Denote trough $\underset{i\in I}{\times} \mathfrak{G}_{\pi_i}$ the class of all groups which are the direct products of their Hall π_i-subgroups. This class is a particular case of saturated lattice formations (see [2] or [3]). The main goal of this work is to extend Baer's result for such classes.

According to p.389 of [4] for a local formation $\mathfrak{F}$ the $\mathfrak{F}$-hypercenter $Z_\mathfrak{F}(G)$ of a group G is the maximal F-hypercentral normal subgroup for the canonical local definition F of $\mathfrak{F}$. As there was shown for any group G the $\mathfrak{F}$-hypercenter exists and is unique. In particular when $\mathfrak{F}$ is the class of all nilpotent groups we have $Z_\infty(G) = Z_\mathfrak{F}(G)$.

Our main result is

Theorem 2. *Let $\sigma = \{\pi_i | i \in I$ and $\pi_i \cap \pi_j = \emptyset$ for all $i \neq j\}$ be a partition of non-empty set of primes π and $\mathfrak{F} = \underset{i\in I}{\times} \mathfrak{G}_{\pi_i}$. If G is a π-group then:*

(1) A π_i-element g of G belongs to $Z_\mathfrak{F}(G)$ iff g permutes with all π_i'-elements of G.

(2) The intersection of normalizers of all maximal π_i-subgroups of G for all $i \in I$ is $Z_\mathfrak{F}(G)$.

References

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