Finite soluble groups in which all n-maximal subgroups are $\mathcal{F}$-subnormal

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Throughout this paper, all groups are finite. We use $\mathcal{U}$, $\mathcal{N}$ and $\mathcal{N}^r$ to denote the class of all supersoluble groups, the class of all nilpotent groups and the class of soluble groups of nilpotent length at most r ($r \geq 1$).

Recall that a subgroup H of G is called a 2-*maximal* (*second maximal*) subgroup of G whenever H is a maximal subgroup of some maximal subgroup M of G. Similarly we can define 3-*maximal subgroups*, and so on. Let $\mathcal{F}$ be a non-empty formation. Recall that a subgroup H of a group G is said to be $\mathcal{F}$-*subnormal in* G if either $H = G$ or there exists a chain of subgroups $H = H_0 < H_1 < \ldots < H_n = G$ such that H_{i-1} is a maximal subgroup of H_i and $H_i/(H_{i-1})_{H_i} \in \mathcal{F}$, for $i = 1, \ldots, n$.

We study groups in which all n-maximal subgroups are $\mathcal{F}$-subnormal. The following theorems are proved.

Theorem A. *Let $\mathcal{F}$ be an r-multiply saturated formation such that $\mathcal{N} \subseteq \mathcal{F} \subseteq \mathcal{N}^{r+1}$ for some $r \geq 0$. If every n-maximal subgroup of a soluble group G is $\mathcal{F}$-subnormal in G and $|\pi(G)| \geq n + r + 1$, then $G \in \mathcal{F}$.*

Theorem B. *Let $\mathcal{F} = LF(F)$ be a saturated formation such that $\mathcal{N} \subseteq \mathcal{F} \subseteq \mathcal{U}$, where F is the canonical local satellite of $\mathcal{F}$. Let G be a soluble group with $|\pi(G)| \geq n + 1$. Then all n-maximal subgroups of G are $\mathcal{F}$-subnormal in G if and only if G is a group of one of the following types:*

I. *$G \in \mathcal{F}$.*

II. *$G = A \rtimes B$, where $A = G^{\mathcal{F}}$ and B are Hall subgroups of G, while G is Ore dispersive and satisfies the following:*

(1) *A is either of the form $N_1 \times \ldots \times N_t$, where each N_i is a minimal normal subgroup of G, which is a Sylow subgroup of G, for $i = 1, \ldots, t$, or a Sylow p-subgroup of G of exponent p for some prime p and the commutator subgroup, the Frattini subgroup, and the center of A coincide, while $A/\Phi(A)$ is an $\mathcal{F}$-eccentric chief factor of G;*

(2) *every n-maximal subgroup of G belongs to $\mathcal{F}$ and induces on the Sylow p-subgroup of A an automorphism group which is contained in $F(p)$ for every prime divisor p of $|A|$.*

Theorem C. *Let $\mathcal{F}$ be a saturated formation such that $\mathcal{N} \subseteq \mathcal{F} \subseteq \mathcal{U}$. If every n-maximal subgroup of a soluble group G is $\mathcal{F}$-subnormal in G and $|\pi(G)| \geq n$, then G is ϕ-dispersive for some ordering ϕ of the set of all primes.*