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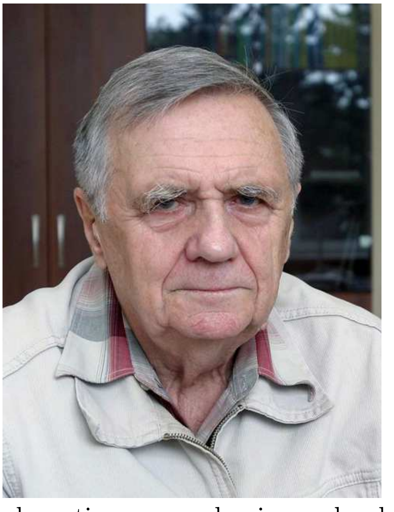
MATHEMATICAL LIFE

Sergei Konstantinovich Godunov has turned 85 years old



Academician Sergei Konstantinovich Godunov, one of the brightest stars of Russian and international computational science in the 20th century and an acknowledged leader in modern applied mathematics, turned 85 in 2014. A researcher with a broad spectrum of interests, he has made fundamental contributions to the development of computational mathematics, numerical methods in continuum mechanics, and the theory of quasi-linear differential equations. His investigations have played a basic role in the formation and evolution of several areas of mathematics, such as well-posedness of boundary-value problems for differential equations, difference schemes, and numerical methods of linear algebra, algorithms for solving problems in gas dynamics and continuum mechanics and calculating the deformations of metals under explosive forces, and the guaranteed accuracy of numerical computations. His works were fundamentally important for the solution of the problem of the use of nuclear energy, the realization of the Soviet Atomic Program, and the development of a hydrogen bomb. Godunov's pioneering research has to a significant degree shaped the modern appearance of numerical analysis and the most prospective trends of its development. It is characteristic of his investigations that they systematically combine abstract mathematical thinking with the ability to solve concrete applied problems. His name is inseparably connected with several important achievements in mathematics and with numerical methods such as the method of decay of discontinuities (widely known as Godunov's method), the establishment method in flow problems, and the method of mathematical modelling of elastoplastic deformations. He discovered a connection between the well-posedness of equations of mathematical physics and thermodynamics, based on the modern extended formalization of the latter. Godunov's approach to research includes some basic principles which are crucially important for understanding the true nature of solutions to problems, in particular, in shock-wave or explosive processes, as well as in models of micro- and nanopowders. In his works he developed

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a new mathematical apparatus for investigating problems in continuum mechanics, and he proposed new tools in the form of algorithms and software, namely, special-purpose libraries of applied programs. Godunov formulated the concept of guaranteed accuracy in numerical analysis, thanks to which new fundamental notions were introduced in the routines of computational mathematics, for instance, the spectral portrait of a matrix, the performance criterion for dichotomy, and the generalized Lyapunov equation.

Sergei Godunov was born in Moscow on 17 July 1929, in a family of representatives of the Russian intelligentsia. His father, Konstantin Dmitrievich Godunov, was a graduate of the Kachinskoe Pilot School and then, together with other first graduates, the Zhukovsky Air Force Engineering Academy. He was a well-known Russian aeronaut, a designer of aerostats, and a participant of the record-breaking flight on the USSR-1 stratostat on 30 September 1933. Sergei's mother, Ekaterina Viktorovna Godunova, neé Chertova, was a graduate of the Moscow Institute for Noble Girls (a finishing school) and Dijon University (in France). She was a philologist, a teacher of the German and French languages (and literatures). Sergei Godunov has recollected that in his childhood he constantly felt the influence of the three languages Russian, French, and German, and nowadays he understands German quite well and is fluent in French.

On graduating from the 1st Moscow School of the Air Force in 1946, which he had attended since 1944, Godunov continued his education at Moscow State University. Beginning from the first months of his study in the Faculty of Mechanics and Mathematics, he plunged enthusiastically into research under the supervision of B. N. Delaunay, a corresponding member of the USSR Academy of Sciences. As a student he also participated actively in the work of a seminar supervised by A. O. Gel'fond and A. Ya. Khintchine, both corresponding members of the Academy of Sciences. A recipient of a Stalin stipend since his second year, Godunov solved a serious problem in the theory of continued fractions. This result of his was presented for publication in *Doklady Akademii Nauk* by Academician I. M. Vinogradov and was published in 1948 [1]. At Moscow State University Godunov was educated in the scientific schools of Academicians I. M. Gelfand and I. G. Petrovskii. He prepared his diploma thesis under Petrovskii's supervision and graduated from the university in 1951, receiving a diploma with distinction in the specialty of computational mathematics. Academician M. V. Keldysh was an official opponent at his thesis defence. Starting in 1952, Godunov taught at Moscow State University, as an assistant professor in the Department of Differential Equations.

In 1954 he finished his postgraduate studies at the Steklov Mathematical Institute of the Academy of Sciences, where Petrovskii was his scientific advisor. The same year he was awarded a Ph.D. degree in computational mathematics (the topic of his dissertation was classified; Academician S. L. Sobolev and Professor N. N. Meiman were official opponents).

It turned out that Boris Delaunay did not just entice Godunov into fundamental research, but also made him keen on long weekly hikes in the countryside and then also on mountain hiking. Godunov has kept up these habits for decades, taking part in trips across the Siberian taiga, canoeing on taiga rivers, and hiking in the Tian Shan mountains and in the Pamir foothills. In 1958, while climbing in the Caucasus Mountains, he received a serious head injury in a rockfall, and lost his memory for two weeks. After that he actually had to relearn how to walk. Luckily, he overcame almost completely the effects of that injury and was able to continue his research and teaching at Moscow State University.

Godunov became an associate professor in the Department of Differential Equations in 1965, the same year he received his D.Sc. degree on the basis of his aggregate publications. In 1968 he became a professor in the same department, and in 1976 he was elected a corresponding member of the USSR Academy of Sciences, in the Department of Mathematics. Since 1994 he has been an academician of the Russian Academy of Sciences.

Godunov began working in 1951 at the Steklov Mathematical Institute. From 1953 to 1966 he was a junior researcher, then a researcher, and then a senior researcher in the Department of Applied Mathematics at the institute. From 1966 till 1969 he was a department head at the Institute of Applied Mathematics of the Academy of Sciences. All this while teaching in Petrovskii's department at Moscow State University from 1952 till 1969, first as an assistant professor, then an associate professor, and finally a professor.

In 1969, on an invitation from Academician M. A. Lavrentiev, Godunov moved to Novosibirsk, where he headed a laboratory at the Computer Centre of the Siberian Branch of the Academy of Sciences. In 1980 he moved to the Institute of Mathematics of the Siberian Branch (now the Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences), where he was a department head, a deputy director, the acting director (1981–1986), and a councillor of the Academy of Sciences.

It should be mentioned that in 1960 Godunov began regular visits to Akademgorodok in Novosibirsk. At Sobolev's request he gave the lecture course "Difference methods for solving the equations of gas dynamics" [2] in the Faculty of Mechanics and Mathematics of the university, and in August of 1963 he participated in the Soviet–American Symposium on Partial Differential Equations, where R. Courant, K. O. Friedrichs, and P. D. Lax were present.

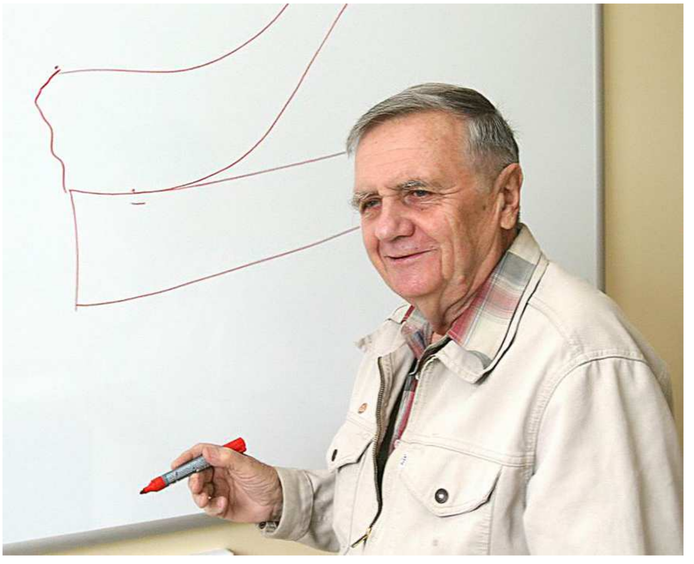
In the present era of narrow specialization, Academician Godunov is a striking example of a scholar who works with equal success in diverse areas of fundamental mathematics and its applications. Continued fractions, differential equations and difference schemes, computational linear algebra, gas dynamics and continuum mechanics — this list of fields where his works have made significant contributions is far from complete.

Turning to applied problems at the suggestion of Keldysh and Gelfand, Godunov proposed in 1954 a scheme for the numerical computation of one-dimensional gas dynamics problems with discontinuous solutions — shock waves. The scheme is based on the use of solutions to the Riemann problem on the decay of discontinuities, and it has an intuitive physical interpretation: a calculation in this scheme reduces to a computation of the decay of a discontinuity followed by averaging over each mesh interval. One starts from this physical interpretation in constructing a scheme for non-linear equations. The most widely known and still often cited paper of Godunov, "A difference method for the numerical calculation of discontinuous solutions of the equations of hydrodynamics" [3], was published in *Matematicheskii Sbornik* in 1959. A thorough account of all the circumstances around the creation of this scheme and the subsequent stages of its development was the subject of Godunov's lecture in Ann Arbor, Michigan (USA) at the symposium "Godunov's method in gas dynamics" in May of 1997. The full text of this lecture was published in [4] and an abridged English translation of it in [5]; a full English translation was published as the preprint [6]. The difference scheme created by Godunov for the calculation of discontinuous solutions of the equations of gas dynamics using the method of shock capturing, which involves an adequate 'smearing' of shock waves, became internationally known as Godunov's scheme. It is currently a standard tool of the numerical analysis of problems in continuum mechanics.

Godunov has stressed that in the creation and revision of his scheme he always worked with others: his teachers, his colleagues, and then also his students. New results usually emerged in the course of discussions and arguments, in comparing the results of numerical computations with actual experiments they performed, and also in the planning of further actions. His initial acquaintance with numerical techniques occurred under the direct influence of K. A. Semendyaev and A. I. Zhukov, while he refined the mathematical formulations of the problems under the influence of Gelfand. Extremely important were the numerous discussions with outstanding physicists such as Academicians Ya. B. Zel'dovich, A. D. Sakharov, and Yu. B. Khariton and Professor D. A. Frank-Kamenetskii, as well as with the team of experimental physicists led by professor L. V. Al'tshuller. Godunov regarded as particularly important his participation in the comparison of numerical computations with experiments. He also was personally involved in the preparation of the experiments and the derivation of the equations of state from the experimental data. The lectures of Gelfand [7] served as an impetus which led him to connect the performance of a discrete model (Godunov's scheme) with the laws of thermodynamics, and in particular, with the law of increasing entropy [8].

A characteristic feature inherent to many non-linear processes is that the solutions of the corresponding equations have singularities which are a priori unknown. A typical example is the gas dynamics model described by a system of quasi-linear hyperbolic equations. Weak discontinuities of solutions of this system propagate along the characteristics at a finite velocity. However, in contrast to the linear case the characteristics can intersect in the process of evolution, and this results in the formation of strong discontinuities: shock waves. A significant part of the information about the details of this formation is lost in the course of time, which can be interpreted as a consequence of the increase of entropy. This common feature of gas dynamics problems is also connected with the fact that specific approaches, including numerical approaches, are required in order to construct their discontinuous solutions.

One of the main impediments in the development of numerical methods for solving gas dynamics problems is the acutely felt lack of theoretical representations and the poor development of a mathematical language for describing the behaviour regimes of solutions of quasi-linear hyperbolic equations at the moment when singularities arise (there is no a priori information about the smoothness structure of a generalized solution [9]). This sharply reduces the value of the numerical methods based on formal and essentially arbitrary approaches to the numerical analysis of gas dynamics problems. This is the weakest link in the chain of computational problems arising here: even the simplest, one-dimensional, gas dynamics problems turn



out to be surprisingly complicated mathematically when one attempts to solve them numerically on a computer. This was the subject of Godunov's preprint [4] and its sequel [10], where he presented striking examples in which a numerical method has a fractional exponent of convergence. Typical numerical examples show that the convergence of the method is ensured for all conservation laws. However, its rate of convergence does not necessarily coincide with the order of accuracy of the difference schemes used for the simulation. For example, for a Godunov scheme with first order of accuracy the errors decay like $h^{0.8}$ in most of the quantities sought, while for the entropy the errors decay like $h^{0.5}$ [10]. As of today, even in the indicated simple problems, let alone more complicated cases, there is little hope for a coherent understanding of the structure of their generalized solutions.

In view of the abundance of *ad hoc* heuristic formulae devised by analogy with one-dimensional gas dynamics and often simply copying it with hopes pinned on the power of modern computers, the existing literature is not very satisfactory. Just one thing is quite clear here: one cannot hope to get an idea of how to organize a numerical method for finding discontinuous solutions of gas dynamics problems (which are precisely the solutions of any practical interest!) in a rational way without an intellectual analysis of the problems under consideration. The issue of the 'regularity' of generalized solutions remains one of the most difficult subjects in these and similar problems, and this also explains why an intellectual analysis, which can reduce the degree of arbitrariness in the choice of an adequate method for numerical solution, is so important in problems of this type. It is for this reason that the numerical construction of discontinuous solutions of the equations of gas dynamics remains a fundamental and still deeply intriguing mathematical problem.

In this connection, each case where information about the well-posedness class of a problem actually contributes to improve the qualitative characteristics of a numerical method is always an event of its own. Godunov's papers [3], [8], [11] were examples of such extraordinary events: they turned out to have a decisive influence on the whole subsequent development of numerical methods for gas dynamics problems. In these papers he proposed new mathematical ideas which underlie the contemporary trends in the construction of numerical methods not only for gas dynamics, but also for many problems in continuum mechanics. Maybe this is why the idea of using the method of exact solution of the Riemann problem on decay of discontinuities in calculations of complex gas dynamics flows — now known worldwide as Godunov's method — initially came as a real surprise.

Godunov was the first to propose a conceptual basis for solving the problem of the numerical determination of discontinuous solutions of the equations of gas dynamics. His method has had a profound effect on the subsequent development of numerical methods and is now a standard tool in the numerical investigation of problems in continuum mechanics, as well as an integral part of the mathematical culture and a kind of international mathematical brand.

He has had many successors trying to refine his scheme, make it more technologically effective, and improve its order of accuracy [12]–[22]. Regarding the important issue of orders of accuracy of the scheme, he himself holds a view different from the views of most of his followers: he believes that the order of accuracy of a scheme must be determined experimentally, on the basis of the weak convergence of conservation laws, as in [10]. The main reason is that there still exists no authoritative theory of generalized solutions of sufficiently general quasi-linear hyperbolic systems of equations.

Godunov thinks that solutions of existing discrete models (difference schemes) should be investigated by taking decreasing mesh spacing and using methods of statistical physics involving Gibbs sums and potentials. Such an approach could be a development of J. Glimm's well-known paper [23], where he justified Godunov's scheme in the one-dimensional case using statistical averages. However, instead of averaging the values obtained in solving the Riemann problems on random rays as proposed in [23], one must average the full solutions of the problem over all the cells in the model.

In describing his views on the logical foundations of the construction of his schemes, Godunov has also sketched the requisite directions of future investigations. On the one hand, the basic principles underlying the difference schemes to be constructed must be further clarified, and the convergence of the approximate solutions obtained from them as the mesh size becomes finer must be studied experimentally. On the other hand, once this convergence has been established, the results of calculations must be compared with experiments. Here one must keep in mind that for such a comparison the equation of state and dissipative coefficients of the medium for which the computations are carried out must be meticulously adjusted. In Godunov's opinion, not only computational mathematicians, but also experts in the relevant areas of physics and mechanics, theorists and experimenters alike, must participate in such an analysis.

The procedures for the numerical analysis of the rate of weak convergence in Godunov's method were worked out by Godunov and Ryabenkii in their joint investigations in the late 1950s. However, the authors confined themselves to oral communications and have never published their results. Subsequently, their ideas were partially used in [10].

The influence of Godunov's innovative approaches to gas dynamics problems has not been limited to the investigations described above. Starting in 1955 he undertook the construction of a two-dimensional version of his scheme. In doing this he had to deviate slightly from the discrete model which had led to the one-dimensional scheme. The first difference was to use the Euler coordinates instead of the Lagrange coordinates. The second difference was that the computation of a single time step could no longer be viewed as a precise model of a physical process. Therefore, in the construction of the new difference scheme one disregards the complex picture of 'two-dimensional' decays of discontinuities at the nodes (where four cells of the mesh adjoin), and uses only the solutions of the one-dimensional Riemann problems on the edges of the cells. The interaction of cells adjoining at corner points is neglected. Using this simple idea and an experimental adjustment of the scheme of calculations, he selected the coefficients for computing one time step so as to obtain the solution of the Riemann problem as a result. Although this approach could not ensure that all the conservation laws held precisely, the entropy was nevertheless increasing: negligibly along smooth solutions but quite strongly, as it should, on discontinuities. This method proved to be successful and has been widely used in practice since then.

In his diploma thesis, G. P. Prokopov, a student of Godunov, applied a modified version of the two-dimensional Godunov scheme to the numerical solution of a flow problem posed by Academician G. I. Petrov. Subsequently Godunov, in conjunction with K. A. Bagrinovskii, G. B. Alalykin, A. V. Zabrodin, and Prokopov, modified his scheme for two-dimensional gas dynamics models with movable grids. In 1961, using this modified scheme and following a suggestion of Petrovskii, people carried out calculations of a stationary transonic flow about an axially symmetric body using the establishment procedure of a non-stationary flow [24]. This establishment method, together with the difference scheme used in it, was universally recognized and has been widely used both in Russia and abroad.

The two-dimensional Godunov scheme has also been generalized to three dimensions, where the meaning of the 'Riemann problem' is problematic. In most applications of this scheme all the energy conservation laws in each cell and at each time step are computed using a linearized isentropic model, while the final computations of energy conservation are made using non-linear equations, which results in an increase of entropy.

The two-dimensional difference scheme in [24] has only been used intensively starting in 1969, first at the Central Institute for Aviation Motor Building [25], [26] and then at other institutes and corporations in Russia and abroad [27].

The 1976 monograph [28] was an important summary of the many years of triumphant applications of the Godunov scheme in its original form. He wrote it in conjunction with his students Zabrodin and Prokopov and his colleagues A. N. Kraiko and M. Ya. Ivanov from the Central Institute for Aviation Motor Building. In 1979 it was translated into French by the "Mir" publishers [29]. Many significant details used in the construction of two-dimensional difference schemes were cleared up by Godunov in his talk at the University of Michigan in 1997 [4].

His investigations concerning the numerical search for discontinuous solutions of the equations of gas dynamics spurred the systematic analysis of the systems of quasi-linear hyperbolic equations which can be expressed as conservation laws. The cornerstones of contemporary numerical methods for hyperbolic equations, including the hyperbolic conservation laws of continuum mechanics, were laid in his papers of the 1950s–60s. Equations in this class have a complicated structure which is difficult to comprehend at first glance.

Godunov concentrated primarily on expressing quasi-linear differential equations in a form compatible with the requirements of continuum mechanics. His many discussions with physicists and also Gelfand's lectures on quasi-linear equations [7] contributed to the establishment of the very important connection between the discrete model (the Godunov scheme) and the laws of thermodynamics. This was the start of a systematic study of the well-posedness of boundary-value problems in continuum mechanics that can be expressed as conservation laws [8], [9], [11], [30], [31]. Chronologically, the first paper in this cycle was [8]. Results there had to do with the connection between the Carnot cycle and the stability of the oscillation processes in the corresponding thermodynamical model. Godunov found a key link between the theorem on a universal integrating factor in the thermodynamic identity and the stability of equilibria of thermally homogeneous dynamical systems with finitely many degrees of freedom. Relying on continuum mechanics models expressed as systems of equations in the form of conservation laws, he proposed that the well-posedness of a boundary-value problem for the equations of continuum mechanics is related to the stability of an equilibrium of a system of ordinary differential equations. This observation turned out to be quite important for the mathematical formulation of the laws of thermodynamics in a form which is convenient for modelling various problems in continuum mechanics.

His attempt at adapting the structure of the equations to the need for an effective organization of computations has led to a certain mathematical formalization of the laws of thermodynamics (and in particular, the law of increasing entropy) with the help of certain customized generalized thermodynamic potentials. In their new mathematical formulation, the equations are taken to a particular conservative (divergent) form more convenient for their theoretical and numerical analysis, and the laws of thermodynamics hold simply due to the structure of these equations, where the additional law of conservation of entropy is also satisfied. The simplest version of such a structural systematization of the equations of gas dynamics was described in [11].

The systematization in [11] was the result of an analysis of the computational practice used in solving gas dynamics problems, carried out with the object of understanding why the laws of thermodynamics actually ensure the reliability of numerical results. It turned out that the indicated generalized thermodynamic potentials could also be convenient for representing other equations of mathematical physics. It should be mentioned that the form of the equations of continuum mechanics proposed by Godunov can be supplemented by terms describing the energy dissipation in the sense of the Onsager theory [32]. This simplifies the analysis of irreversible processes in a continuous medium and, in particular, the investigation of the structure of shock waves — discontinuous solutions of the problems under consideration [9], [11], [30].

In connection with approaches to the investigation of generalized solutions of quasi-linear hyperbolic equations on the basis of the new notion of a generalized thermodynamic potential, the important phenomenon was discovered of the sharp dependence of solutions on the introduction of small dissipative terms into the system. Various small 'viscosities' turned out to lead to various kinds of prohibition of discontinuous solutions for quasi-linear hyperbolic systems. This underlies the following conclusion, which is important in practice and, in particular, in computations: small dissipative terms in quasi-linear equations can result in different solutions, that is, can lead to the non-uniqueness of solutions, although the conservation laws will be satisfied. This is all the more important since unremovable errors (dissipations) always occur in the discrete modelling of physical processes, even when we neglect the physical dissipations occurring in the medium (such as viscosity, thermal conductivity, and the like). Examples of non-uniqueness of solutions of problems of this type were given in [9], [11], and [30]. The first of these papers was reported (before being published) in a seminar at Moscow State University in 1960, in the presence of R. Courant and P. D. Lax, who were then visiting Moscow for the first time. Godunov discussed his ideas about the influence of these non-uniqueness examples on the structural systematization (classification) of boundary-value problems for quasi-linear hyperbolic equations in his survey lecture at the 4th All-Union Mathematical Congress [31].

The paper [11] marked the start of Godunov's multi-year investigations of the connection between the laws of thermodynamics and the well-posedness of the equations of mathematical physics and continuum mechanics. In particular, he introduced in [11] the important class of so-called doubly divergent systems, which contains the system of equations of gas dynamics. Doubly divergent systems have a remarkable property: all the equations in the system can be written in a divergent form so that the system becomes symmetric t-hyperbolic in the sense of Friedrichs. Its coefficient matrix is formed by the second derivatives of the generalized thermodynamic potentials of the medium. When quasi-linear hyperbolic systems expressing conservation laws are written in such a form, there is always a consequence in the form of the entropy conservation law, and this makes it possible to write them in a special divergent form, the Godunov form. This approach enabled Godunov to give a description of a generalized solution in which the entropy production along a smooth solution is zero, while along a discontinuous solution it satisfies a certain inequality (the entropy increment must always be positive). Only a decade later, in 1971, the class of doubly divergent systems was re-discovered by Friedrichs and Lax [33]: they called equations of this type 'systems of conservation laws with convex extension'. The formalism of doubly divergent systems has found broad applications to various problems in continuum mechanics and is now also used in papers on 'extended thermodynamics' [34]–[36].

Not all equations of continuum mechanics can be taken to the required doubly divergent form with the use of generalized thermodynamic potentials. For instance, the equations of magnetohydrodynamics or of elasticity theory are not covered. The papers [37]–[39] were concerned with this difficulty; they are surveyed in [40]–[42]. As a solution, it was proposed to look at an extended system, supplemented by equations compatible with the original system. Then it is necessary to classify all such extensions. A classification of equations on the basis of the Galilean invariance of the problems under consideration was proposed in [43]–[45], where conditions on admissible generating thermodynamic potentials were also imposed, and the equations were written in a form containing special matrices of Clebsch–Gordan coefficients (used in the decomposition of a product of irreducible representations of the rotation group into a sum of irreducible representations). The classification of the equations of continuum mechanics was continued in [46]–[49].

Godunov devoted much time and effort to establishing connections between thermodynamics and well-posedness of general problems in mathematical physics and to finding the place of the equations of continuum mechanics in the theory of quasi-linear hyperbolic equations in the conservative form [28], [50], [51].

Already in [31] he actually advanced the idea of a strong connection between the laws of thermodynamics and the structure of the equations describing the motion of a continuous medium. Subsequently this basic idea led to the notion of 'thermodynamically consistent hyperbolic conservation laws', which govern the motion of a continuous medium and the thermodynamic identities. On this basis Godunov described the structure of conservation laws and classified the equations of continuum mechanics.

Further investigations by him and his students led them to single out the class of 'thermodynamically consistent systems of equations' [40]–[42]. It was also shown that an additional conservation law appears as a necessary condition of compatibility [52]. Such an additional law can be written as a partial differential equation, and it determines the change of entropy in the process of deformation of the continuous medium. It was a remarkable consequence of the proposed form of such equations that they can be represented as symmetric t-hyperbolic Friedrichs systems [11], [47], [53], [54]. In the framework of this theory, questions connected with the formulation of principles of mathematical modelling of various mechanical processes in continuous media, and also questions related to the design of adequate numerical discretizations, are solved in an elegant way from a unified standpoint. In particular, many classical equations of mathematical physics and continuum mechanics are thermodynamically consistent, including the equations of gas and fluid dynamics, magnetohydrodynamics, and relativistic hydrodynamics, the equations of elasticity, the Maxwell equations, and so on [43], [47], [53], [54]. In the general case Godunov's approach provides a key to understanding the actual structure of solutions in non-linear models of continuum mechanics and, in particular, solutions in elastoplastic models of deformations of media, in shock-wave and explosive models of processes, and in models of micro- and nanopowders.

A large family of continuum mechanics models (gas dynamics, non-linear elasticity, Maxwell viscoelastic material, and some others) have a common property: they are described by systems of quasi-linear hyperbolic equations. On this basis Godunov carried out a deep investigation of discretization questions for these models. The fact that a system of equations can be represented in two forms, the conservative form and the symmetric t-hyperbolic form, makes it possible to discretize the problem in two steps, following the predictor-corrector scheme. In the first step, t-hyperbolicity (that is, local (in time) solvability of the Cauchy problem) is used to form a predictor for calculating the flows in the conservation laws. In the second step a corrector is formed using the conservative representation of the equations (in the form of conservation laws). The correction is the crucial step, because at this stage the additional conservation laws and the law of increasing entropy are taken into account (by solving overdetermined systems of partial differential equations). These computational techniques can be applied to the overdetermined thermodynamically consistent systems of equations whose generalized solutions were defined in [31]. We must point out that discussions on the legitimacy of these techniques are still ongoing [10].

Godunov has repeatedly returned to investigations of the connection between the laws of thermodynamics and the well-posedness of equations; his latest results in this direction were published in [54]–[57].

This approach, developed fruitfully by Godunov and his students, led to the development of a new model of non-linear elastoplastic deformations of a continuous medium. The active work on this model was carried out in close collaboration with colleagues occupied with explosion processing of materials (metals) at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences. Under intensive stresses metals behave like fluids. This observation, in combination with Frenkel's results [58] on the dependence of the stress relaxation time on the state of the continuous medium, enabled Godunov to develop a non-linear relaxation model of inelastic deformations. His analysis of large deformations in elasticity and results on plasticity modelling using the non-linear Maxwell viscoelastic model were published in [40]–[42]. Estimates for the parameters characterizing the magnitude and the role of dissipation processes which should be taken into account on the basis of Maxwell's relaxation equations, were obtained in experiments [59]–[62]. The question of how viscous terms must actually be introduced into the model was explained in [59], [61], [63], and [64]. The final version of the model was proposed by Godunov in the joint paper [65]. A cycle of works verifying the Maxwell non-linear relaxation model thus constructed and calculating shock-wave propagation and damping was carried out in the 1970s with Godunov's active involvement. Also in that period, the problem of a submerged jet [66] and the problem of inverted jet formation were solved [67].

In the 2000s Godunov and his students completed an investigation of questions arising in the representation of the equations of non-linear elasticity as a hyperbolic thermodynamically consistent system. In particular, they dealt with the problem of the convexity of the elastic energy with respect to the state parameters of the medium [47], [53]. The non-linear Maxwell model of inelastic deformations is still popular with researchers in this country and abroad. It is also used in computations of extremal deformations in problems of high-velocity collision and high-energy impact on metals [34]–[36].

During one of Godunov's trips to Akademgorodok in Novosibirsk, Lavrentiev drew his attention to the problem of explosion welding. The question was about the causes for the undulation observed at the interface between two metallic plates being welded together as a result of collision. This collision occurs as the plates are accelerated into each other by detonations of explosive charges. By analogy with the then existing theory of jet formation due to Lavrentiev, it was assumed that an ideal fluid model is suitable for describing the collision of metal plates in the context of explosion welding. It was believed that in the collision of the plates a cumulative jet of metal is formed which moves more rapidly than the point of contact of the plates. For all this, in most experiments no jet was observed, and this was explained by jet breakdown due to instability. However, the first calculations of the collision of plates in acoustic and hydrodynamic approximations carried out in 1968–69 by a Godunov-led team, found no such jet. The conjecture that conservation laws only held with error in these calculations turned out to be unfounded: the difference scheme taken ensured the unconditional satisfaction of all the conservation laws. A comparison of the results of calculations with experimental data revealed that, instead of a cumulative jet, a 'submerged jet' formed near the weld interface, and it had a lower velocity than the nearby material of the plates being welded. It was concluded that the absence of a cumulative jet in the real process was due not to its instability but to the presence of viscosity in the materials of the plates being welded. To substantiate this conclusion, Godunov proposed a mathematical model of non-linear inelastic deformations of solids under conditions of a high-velocity deformation of the medium, based on the Maxwell model of stress tensor relaxation. As a result of calculations carried out by Godunov and his students for oblique collisions of metal plates, they discovered a viscous submerged jet which could be observed experimentally.

Among other things, this mathematical model created with the help of Godunov made it possible to develop a new method for measuring the viscosity of metals undergoing high-velocity deformation. In 1972 these groundbreaking works were honoured by the A. N. Krylov Prize of the Academy of Sciences, and they were singled out as extremely important in the survey [68] on shaped charges.

At the suggestion of Academician V. M. Titov, Godunov offered the lecture course "Elements of continuum mechanics" [42], based on these investigations, in the Faculty of Physics at Novosibirsk University. In 1993 this course was honoured by the M. A. Lavrentiev Prize of the Russian Academy of Sciences. An extended and revised version of these lectures, co-authored with Godunov's student E. I. Romensky, was published first in Russian [40] and later in English [41].

Godunov created a mathematical model of viscoelastic deformations of metals in the intermediate zone between the domains of applicability of the purely elastic approach and the gas dynamics approach [65], [69]–[71], and he is also the author of a non-linear relaxation model of elastoplastic deformations [42]. In subsequent joint papers with his students the model of elastoplastic deformations was applied, in particular, to the investigation of non-linear deformations [46]–[49], [53], [72].

The development of the non-linear Maxwell relaxation model led to a theory of plastic deformations of a medium, an outline of which was sketched in [62]. Further results, concrete calculations, and experimental data were published in [61], [73]. The theory thus developed has prompted a natural desire to apply it to the analysis of the undulation in explosion welding. It turned out that it is just the residual stresses that are responsible for this undulation. The existence of residual stresses acting in a compressive fashion on the boundary layer in a metal plate subjected to a stationary moving momentum of pressure has been confirmed numerically [74]. Special explosion experiments at the Lavrentiev Institute of Hydrodynamics of the Siberian Branch of the Russian Academy of Sciences also confirmed this effect by finding small 'wavelets' along the weld interface of the plates.

It was established in subsequent investigations that the undulation on the contact interface between the plates was produced by self-excited oscillations of the medium in a neighbourhood of the point of contact. Self-excited oscillations occur in a neighbourhood of the high-pressure domain in the zone of viscoelastic deformations of metals which is intermediate between the domains of applicability of the purely elastic approach and the gas dynamics approach. This neighbourhood is a kind of 'resonator', responsible for the further development of oscillation processes. The first applications of these models to the calculation of deformations under the influence of explosive loads were made in the joint papers [46] and [47]. The results of calculations were published in [53] and reported in Godunov's plenary talk [52] at the Henri Poincaré Institute in Paris (January 2008) and at the International Congress on Computational Fluid Dynamics in St. Petersburg (2010) [75].

Several years ago Godunov returned to the problem of explosion welding, where the question of undulation on the interface under an oblique explosive collision of plates (which had been posed by Lavrentiev) was still open. Experiments showed that physical processes in the contact area occur on the nanostructure level, and that more refined mathematical models based on the methods of molecular dynamics were required for their adequate description. A new mathematical model of non-linear inelastic deformations of solids in the Lagrange coordinates was proposed. Next, a numerical algorithm was designed on the basis of the Godunov scheme and then implemented on a multiprocessor computer, thus making it possible to describe high-velocity interactions of solids. Calculations have confirmed the appearance of undulations on the contact interface between solids under high-velocity oblique collisions [76].

Thus, the results of mathematical modelling of processes in continuum mechanics on the basis of the Maxwell relaxation model and methods of molecular dynamics faithfully reflect, on a qualitative level, the main experimental regularities observed in oblique collisions of plates in explosion welding.

Godunov's work in numerical mathematics has not been limited to the problems mentioned above. Since the early 1950s he has actively studied spectral problems arising in the calculation of critical parameters of nuclear reactors, using the method of spherical harmonics. Together with his student I. A. Adamskaya he created and justified a new technique for the construction of numerical solutions of boundary-value problems for ordinary differential equations, which was called the orthogonal sweep method and has been widely used [77], [78].

In his joint study with Prokopov of spectral problems for elliptic operators, Godunov discovered several computational paradoxes connected with round-off errors in computer calculations that can lead to a computational catastrophe. These results were published in 1970 [79] and were also obtained independently in 1971 by C. C. Page [80], [81], a student of G. H. Golub.

Thus, in computer calculations we must always take round-off errors into account. Numerical algorithms which are satisfactory from the standpoint of theoretical mathematics may well turn out to be ineffective from the standpoint of computer mathematics. Round-offs not only mean loss of the operational properties of real numbers, but also impose stronger requirements on algorithms and on their stability. Without giving due regard to the specifics of computer calculations one can easily err both in choosing the numerical method itself and in evaluating the actual quality of the results of the calculations. In this connection there is now a trend in computer-aided calculations to stress the non-triviality of the effect of round-off errors on the process of computer calculations. As a consequence, some of the traditional mathematical ideas and methods even had to be discarded and replaced by new ones specially tailored to the needs of actual calculations.

The question as to whether round-off errors govern the computational process or vice versa is irrelevant here: there can be no computer calculations without round-off errors (in the arithmetic of real numbers), and on the other hand, there would be no round-off errors without computer calculations. For this reason it is very important to be able to adapt a numerical method to operations on numbers with finitely many digits. In particular, round-off errors in spectral problems in linear algebra often manifest themselves as catastrophic distortions of the results of calculations [82], [83].

Godunov made a thorough study of the question of well-posed formulations of spectral problems for finite-dimensional operators (matrices) from the point of view of computer calculations [83], and this has proved to be a most significant conceptual contribution to the development of new ideas and new computer-adapted statements of spectral problems in linear algebra. He also gave examples of 'pathological' matrices for which the spectral problem is ill-posed.

At the present stage in the development of numerical methods it has become clear that problems to be solved can be 'well-posed' or 'ill-posed'. This quality (or correctness) of the computer formulation of a problem characterizes the stability of its solution with respect to errors in the input data and round-off errors of the arithmetical operations. The quality can usually be expressed by a concrete numerical characteristic, for example, the condition number of the matrix. Thus, an algorithm for the numerical solution of a problem should produce, together with the solution, numerical characteristics of the quality of the statement of the problem. When these characteristics are large, we must be cautious about using the numerical solution obtained: we cannot put much trust in a computer solution if it displays poor stability with respect to errors in the input data or round-off errors.

Questioning the opinion (still existing among practitioners of computation) that the theory and practice of the methods of linear algebra have long been established and have even acquired a kind of exemplary canonicity, Godunov has been pointing out the numerical aspects of this circle of problems. After seriously rethinking the very concept of accuracy of a numerical solution, he introduced into mathematical practice the notion of guaranteed accuracy of computer calculations [81]–[84].

The acceptance of his concept of guaranteed accuracy resulted in abandonment of the use of such classical notions as Krylov subspaces and the Jordan normal form of a matrix, which were replaced by new fundamental notions such as the ε-spectrum, spectral portraits, the performance criterion for the dichotomy and partition of the spectrum, the generalized Lyapunov equation, and some others [83], [85]. Most attention was focused on one-dimensional spectral portraits, which visualize the partition (dichotomy) of the spectrum of a matrix into zones bounded by straight lines, circles, and ellipses [86]–[88]. These new definitions and the associated visualizations were presented by Godunov at the international conference dedicated to the 90th anniversary of the birth of Petrovskii (Moscow State University, 1991) [89]. The theory of ε-spectra and the corresponding computational algorithms are presently being actively developed by L. N. Trefethen in Oxford [90], [91].

We remark that a hint at the introduction of the notions of the ε-spectrum and spectral portrait of a matrix into mathematical practice was given as far back as the joint paper [92] with Ryabenkii. Godunov's lectures [93] at the Universities of Rennes and Brest (prepared with the active involvement of M. Sadkane) promoted the recognition of the practical importance of the notions of the spectral portrait of a matrix and the dichotomy of its spectrum.

These new mathematical notions in computational practice emerged in essence as a particular kind of visualization of the sensitivity of numerical methods to the parameters featuring in Lyapunov's classical stability theorems. For instance, the problem of the stability or instability of the zero solution of a system of linear ordinary differential equations with constant coefficients was solved using these notions, without calculating the eigenvalues of the matrix of coefficients. Furthermore, a new numerical criterion was proposed for estimating the quality of the stability, and on its basis the problem of whether all the eigenvalues of the asymmetric coefficient matrix of the system lie in the left half-plane was effectively solved [88]. Also, the following question was answered in the same way using the same tools: do the eigenvalues of a matrix fall into two groups of those in the left and right half-planes, so that there are no eigenvalues on the imaginary axis [94]? A positive answer obtained using a numerical algorithm means that the partition of the spectrum into the two parts is stable. On the other hand, a negative answer means that some eigenvalue of the matrix or a small perturbation of it can lie on the imaginary axis. Algorithms for the dichotomy of the spectrum of a matrix by a circle, an ellipse, or a parabola have also been proposed [95]. It is extremely important that to answer these key questions in stability theory we need not calculate the eigenvalues of the matrix. In this connection we mention Godunov's lecture "Applications of the new mathematical tool 'one-dimensional spectral portraits of matrices' to the problem of aeroelastic vibrations" at the ISUAAAT-2006 conference in Moscow [96], [97].

Whether problems in mathematical physics are correctly solved depends essentially on the efficiency, well-posedness, and quality of the software available for the solution of basic problems in linear algebra. We remark that, for instance, from the point of view of a theoretical mathematician the solution of a system of linear equations on the basis of Cramer's rule is certainly correct. On the other hand, from the point of view of a computational mathematician Cramer's algorithm alone is not sufficient for computer computations: the person computing also needs stability of the algorithm and manageability of the number of mathematical operations, and Cramer's algorithm has neither of these properties because of effects of machine arithmetic: round-off errors. A cycle of investigations on the corresponding topics centred on the main problems in linear algebra was carried out under Godunov's supervision by a team at the Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences. They focused on the influence on the answer of round-off errors committed in the practical implementation of numerical algorithms. The aim of these investigations was to work out a new and fundamentally important concept, a guaranteed number of accurate decimal digits in the numerical answer. As a result, they obtained fundamental results based on new methods of orthogonal transformations of matrices and vectors using arithmetic with separately stored exponents. Their numerical algorithms always give a definite numerical answer: the user either obtains a result with a guaranteed estimate for the error or gets a motivated rejection of the solution of the problem [82].

The book [82] is one of the most fundamental works concerned with the detailed analysis of the influence of machine round-offs on the accuracy of computation results. As a special feature, this book contains effective estimates for the errors of all the numerical algorithms described in it. Thus, together with the solution of the problem the user always gets corresponding guaranteed estimates for all the errors. With properly organized computations, this enables one to produce computer-assisted proofs, which means that from the standpoint of rigour the numerical answer obtained is a mathematical theorem essentially describing a neighbourhood containing the exact solution of the problem. The proof of the theorem is given by the procedure of computer calculations itself, which is organized in accordance with the concept of guaranteed accuracy.

As a result of all these activities in the investigation of methods for computing solutions of problems in linear algebra, a mathematical apparatus has been developed, and then on the basis of it a specialized library of applied software has been created which takes into account the requirements of computer technologies. The software package PALINA has been designed to solve systems of linear equations and symmetric eigenvalue problems, for singular decomposition of matrices and related problems. It was published before the now popular LAPACK software package.

The papers [86], [98]–[100] by Godunov and some of his students were devoted to finding reasons for many computational paradoxes in linear algebra. Godunov has also made important contributions to the development of the general theory of difference schemes: his 1962 monograph [101] with Ryabenkii is still quite popular (see also [102]). Moreover, since the 1960s he has periodically returned to the fundamental problem of constructing finite-difference grids, because the non-linear nature of the equations under consideration (in the two-dimensional and especially three-dimensional cases) manifests itself first and foremost in its effective constructive solution. The reader can look at the joint papers [103]–[107] regarding the non-triviality of the formulation of this problem even in two dimensions.

Godunov has initiated several new lines of research involving combinations of the theory of hyperbolic equations with hydrodynamics and elasticity theory, difference methods for solving differential equations with mathematical physics, and linear algebra with problems in the thermodynamics of continuous media. His works have determined to a significant degree the modern face of numerical analysis, pointing out the most promising directions for the development of this important branch of mathematics.

His unique research style is distinguished by his remarkable integrity and deep understanding of the essence of the problem under investigation, by the abundance of new ideas in combination with the breadth and depth of his analyses, and his ability to see the influence of the whole of mathematics in each particular problem. His works give the reader a clear understanding of the significance of fundamental mathematics for the applied sciences and its decisive role in forming correct notions about the world. All this is combined with an extraordinary ability to find unifying concepts in mathematics and with a focus on the most challenging problems. His colleagues and students say that, as a researcher, he has always been a step ahead of others.

Godunov's research results have invariably been received with great interest, and moreover by many experts in diverse areas of science, thus reminding us that one should not be confined to the narrow limits of one's specialization. He is firmly convinced that all the substantial progress in contemporary science, including its applied areas, is a result of the use of deep interdisciplinary connections in the form of mutual penetration of ideas and results from the most diverse and often distant areas of fundamental science. This view is supported by the fact that a priority of the present-day scientific community is the education of a cadre of highly qualified specialists in areas of science determining the main directions of future development, such as differential equations, numerical mathematics, and mathematical modelling. Godunov is chairman of a dissertation council which guides the education of such specialists. In the year of his 85th birthday he supervises the work of two internationally well-known research seminars at the Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences: "Mathematics in applications" (see http://math.nsc.ru/seminar/godunov/2014.html) and "Statements of problems which admit parallelization for execution on multiprocessor computer systems" (see http://math.nsc.ru/seminar/parall/2014.html).

Godunov is the author of more than 300 research publications, including 17 monographs and textbooks, some of which have appeared in English. His publications have played an important role in giving a modern form to many branches of the applied sciences. They have not only changed the views on how one should approach a wide range of applied problems, but also provided practical algorithms for their computer-aided solution. A directory of Godunov's publications was compiled in 2009, listing all his works published up to that time [108].

Godunov is not only widely known as a prominent researcher, but also as an excellent teacher. His interesting and deep lectures are characterized by maximal clarity, logicality, and accessibility. In his presentations of almost all topics, even when he follows well-known patterns, he introduces essential improvements which make the material as understandable as possible. In his teaching of mathematics he has been and remains a tireless reformer, and his textbooks on differential equations, continuum mechanics, and linear algebra have long been an indispensable component of mathematics lecture courses at both Moscow and Novosibirsk State Universities. Results from his school of research concerned with well-posedness of the statements of problems for the differential equations of continuum mechanics, numerical algorithms for solving such problems, and numerical methods of linear algebra are internationally known.

Godunov has taught for more than half a century, first at Moscow State University, and then at Novosibirsk State University. He worked at the latter from 1969, on a part-time basis, first as a professor in the Department of Differential Equations and then from 1974 till 1990 as the head of that department. There he gave lecture courses in the Faculty of Mechanics and Mathematics and the Faculty of Physics: "Continuum mechanics", "Equations of mathematical physics", "Methods of approximate calculations", "Differential equations", "Numerical methods of linear algebra", "Modern aspects of linear algebra", "Theory of hyperbolic systems", and "Equations of non-linear elasticity". In 2000 he taught the obligatory masters course "Lectures on modern aspects of linear algebra" in the Faculty of Mechanics and Mathematics.

This course was distinguished by the highest degree of innovation and brilliantly expressed personal approach. It was subsequently taught by other professors there.

The breadth and variety of Godunov's investigations, his enthusiasm, his mastery of the whole mathematical arsenal, his capacity for hard work, and his consistency in standing up for his ideas, views, and principles have attracted talented students to research. In 1975 he began leading a research seminar on hyperbolic equations in his department. Around this seminar a collection of teachers, students, and graduate students formed who were actively interested in problems in mathematical physics. They made up the kernel of Godunov's school of research and pedagogy in Novosibirsk. The group began by studying the well-known papers by H.-O. Kreiss [109] and R. Sakamoto [110] on the well-posedness of mixed problems for hyperbolic equations and systems. The goal of their study was a constructive proof that such problems are well posed, using the machinery of energy integrals [111], [112]. From this viewpoint they investigated mixed problems for the wave equation, the vector wave equation, and the linearized system of the equations of gas dynamics with boundary data on a shock wave [112]–[120]. Besides hyperbolic equations, they intensively studied computational problems in linear algebra prompted by paradoxes in numerical calculations. In particular, for asymmetric matrices they considered the eigenvalue problem connected with deciding whether the zero solution of a system of linear ordinary differential equations with constant coefficients is asymptotically stable [121]. It was required to solve the problem without actually calculating the eigenvalues of the matrix of the system [122]. These studies served as a basis for stating problems in completely new formulations and for creating new computational algorithms for linear algebra [82], [83].

Godunov was a scientific advisor to many students, trainees, and graduate students. One can find his former students, including members of the Russian Academy of Sciences and many holders of Ph.D. and D.Sc. degrees, at diverse universities and scientific centres in the Russian Federation and abroad. Their works have been deservedly recognized by the scientific community, honoured by the Lenin and State Prizes, the Gold Medals for Young Researchers of the USSR Academy of Sciences, and various prizes of the Academy of Sciences named after outstanding scientists of the past.

It is evidence of the international recognition of Academician Godunov's scientific merits that international conferences devoted to his methods have been held outside Russia. The first of these, "Godunov's Method for Dynamics: Current Applications and Future Developments" (Ann Arbor, 1–3 May 1997), took place in the USA, and the second, "Godunov's Methods: Theory and Applications" (Oxford, 12–22 May 1999), in Great Britain.

Godunov is a member of the editorial boards of several scientific journals: *Matematichskie Trudy*,[1] *Sibirskii Matematicheskii Zhurnal*,[2] *Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki*,[3] and *International Journal of Computational Fluid Dynamics*. He was awarded an honorary doctorate from the University of Michigan (1997), he received the prize "For Outstanding Contributions to Research

[1]Translated into English as *Siberian Advances in Mathematics*.
[2]Translated into English as *Siberian Mathematical Journal*.
[3]Translated into English as *Computational Mathematics and Mathematical Physics*.

in the Fields of Mathematics, Mechanics, and Applied Physics" given by the Academician M. A. Lavrentiev Foundation (2005), and he was awarded the distinction "For Services to the Novosibirsk Region" (2004).

For his role in accomplishing special tasks for the government and for the solution of important problems in new defence technologies Godunov was awarded the Lenin Prize in 1959. He was honoured with two Orders of the Red Banner of Labour (in 1956 and 1975), two Orders of the Badge of Honour (in 1954 and 1981), the Order of Honour (2010), the medal "For Valiant Work, In Commemoration of the 100th Anniversary of the Birth of Vladimir Il'ich Lenin" (1970), the medal "Veteran of Labour" (1996).

He was awarded the A. N. Krylov Prize of the USSR Academy of Sciences in 1972 for his works investigating the attendant effects of explosion welding, and he received the M. A. Lavrentiev Prize of the Russian Academy of Sciences (1993) for his book *Elements of continuum mechanics*.

Sergei Konstantinovich met his 85th birthday full of energy and creative plans, as usual. We wish him further successes in research, good health, and many years of inspired creative life to the benefit of Russian and world science. We are sure that for years to come he will make us happy through his brilliant results.

V. N. Belykh, *K. V. Brushlinskii*, *V. L. Vaskevich*,
S. P. Kiselev, *A. N. Kraiko*, *A. G. Kulikovskii*, *V. I. Mali*,
V. V. Pukhnachev, *E. I. Romensky*, *and V. S. Ryaben'kii*

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