# Generating simple groups and their subgroups

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It is well known that every finite simple group can be generated by two elements and this leads to a wide range of problems that have been the focus of intensive research in recent years, such as random generation, $(2,3)$-generation and so on. In this talk I will report on recent joint work with Martin Liebeck and Aner Shalev on similar problems for subgroups of (almost) simple groups, focussing on maximal and second maximal subgroups. In particular, we prove that every maximal subgroup of a simple group can be generated by four elements (this is best possible) and we show that the problem of determining a bound on the number of generators for second maximal subgroups depends on a formidable open problem in Number Theory. I will highlight some applications to primitive permutation groups and subgroup growth. Finally, I will report on some related work in progress concerning a new notion of depth for finite groups, focussing on results for simple groups.

References

[1] T.C. Burness, M.W. Liebeck and A. Shalev, *Generation and random generation: from simple groups to maximal subgroups*, Advances in Mathematics **248** (2013), 59–95.

[2] T.C. Burness, M.W. Liebeck and A. Shalev, *Generation of second maximal subgroups and the existence of special primes*, submitted (arxiv:1611.06196).

[3] T.C. Burness, M.W. Liebeck and A. Shalev, *The depth of a finite simple group*, in preparation.

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