Weakly Locally Projective Graphs and Groups

Alexander Ivanov

I will discuss graphs Γ with edge-transitive automorphism group G, which are (a) bipartite with vertices in one part having valency $\alpha \in \{2,3\}$ (they are called *lines*), and the vertices in the other part having valency $2^n - 1$ for some $n \geq 2$ (they are called *points*); (b) the stabilizer in G of a point induces the natural doubly transitive action of $L_n(2)$ on the set of lines containing this point. The ultimate goal is to classify the amalgams $\{G(p), G(l)\}$, where $\{p, l\}$ is an incident point-line pair. The problem was solved by D.Djoković and G.Miller in 1980 for $(\alpha, n) = (2, 2)$ based on earlier work by W.Tutte (1947) and C.Sims (1968); by D.Goldschmidt in 1980 for $(\alpha, n) = (3, 2)$, by S.Shpectorov and the author in 2002 for $\alpha = 2$ and arbitrary n based on results due to V.I.Trofomov (published in a series of papers through 1990's). The classical examples for $\alpha = 3$ are pointline collinearity graphs of the linear and symplectic dual polar spaces over GF(2); while the exceptional ones are the Cooperstain geometry of $G_2(3)$ and the tilde geometries of M_{24} , He, Co_1 and the Monster group.

IMPERIAL COLLEGE LONDON, UNITED KINGDOM