On isomorphism problem for coherent configurations associated with nonsolvable groups

Andrey Vasil′ev

The coherent configuration $\mathcal{X} = (\Omega, S)$ on a set Ω is said to be schurian, if the set S of its basis relations is precisely the set $\mathrm{Orb}_2(G, \Omega)$ of 2-orbits of some permutation group $G \leq \mathrm{Sym}(\Omega)$, that is the set of the orbits of the naturally induced action of G on $\Omega \times \Omega$. Clearly, in the case of a schurian configuration $\mathcal{X}$, this group G is a subgroup of the automorphism group $\mathrm{Aut}(\mathcal{X})$. So starting with G and trying to find $\mathrm{Aut}(\mathcal{X})$ or the set $\mathrm{Iso}(\mathcal{X}, \mathcal{X}')$ of all isomorphisms from $\mathcal{X}$ to an arbitrary coherent configuration $\mathcal{X}'$, we have an advantage knowing some predetermined information about $\mathrm{Aut}(\mathcal{X})$. The same picture arises in the case of Cayley graphs (or Cayley schemes, which are coherent configurations as well). Indeed, if $\Gamma = \mathrm{Cay}(G, X)$ is the Cayley graph for a group G with a connection set X, then G is included as a regular subgroup in $\mathrm{Aut}(\Gamma)$. We are going to discuss some new results and techniques on the isomorphism problem for combinatorial objects associated with a group G in the described way, concentrating on the cases when G is a nonabelian simple or almost simple group.

Sobolev Institute of Mathematics and Novosibirsk State University, Russia
E-mail address: vasand@math.nsc.ru