Number of Sylow subgroups in finite groups

Evgeny Vdovin

For a finite group G, denote by $\nu_p(G)$ the number of Sylow p-subgroups of G. It is a trivial exercise to check that for every subgroup H of G, the inequality $\nu_p(H) \leq \nu_p(G)$ holds. However $\nu_p(H)$ does not divide $\nu_p(G)$ in general. In 2003 G. Navarro proved that $\nu_p(H)$ divides $\nu_p(G)$ for every $H \leq G$ if G is p-solvable. We prove that $\nu_p(H)$ divides $\nu_p(G)$ for every $H \leq G$ if this property holds for every nonabelian composition factor of G. Thus we obtain a substantial generalization of Navarro's result and also give an alternative proof for Navarro's result.

We say that a group G satisfies $\mathbf{DivSyl}(p)$ if $\nu_p(H)$ divides $\nu_p(G)$ for every $H \leq G$.

Theorem. *Let*

$$1 = G_0 < G_1 < \ldots < G_n = G$$

be a refinement of a chief series of G. *Assume that for each nonabelian factor* G_i/G_{i-1} *and for every* p*-subgroup* P *of* $\mathrm{Aut}_G(G_i/G_{i-1})$, *the group* $P(G_i/G_{i-1})$ *satisfies* $\mathbf{DivSyl}(p)$. *Then* G *satisfies* $\mathbf{DivSyl}(p)$.

References

[1] G. Navarro, *Number of Sylow subgroups in p-solvable groups*, Proc. Amer. Math. Soc. **131** (2003), no. 10, 3019–3020.

Sobolev Institute of Mathematics, Russia
E-mail address: vdovin@math.nsc.ru