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ELLIPTIC EQUATIONS WITH VARIABLE ANISOTROPIC NONLINEARITIES

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We study the Dirichlet problem with zero boundary conditions for the elliptic equations with variable anisotropic nonlinearities

$$-\sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left[a_i(x,u) \left| \frac{\partial u}{\partial x_i} \right|^{p_i(x)-2} \frac{\partial u}{\partial x_i} + f_i(x,u) \right] = f(x,u) \quad \text{in } \Omega \subset \mathbb{R}^n , \tag{1}$$

$$-\sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left[a_i(x,u) |u|^{\alpha_i(x)} \frac{\partial u}{\partial x_i} + f_i(x,u) \right] = f(x,u) \quad \text{in } \Omega \subset \mathbb{R}^n , \tag{2}$$

in a bounded domain $\Omega \subset \mathbf{R}^n$ with Lipschitz-continuous boundary $\Gamma = \partial\Omega$. It is assumed that $p_i(x) \in (1,\infty)$, $\alpha_i(x) \in (-1,\infty)$ are given functions such that $p_i(x) \in C^0(\overline{\Omega})$ with the logarithmic module of continuity. Equations of these types emerge from the mathematical modelling of various physical phenomena, e. g., processes of image restoration, flows of electro-rheological fluids, thermistor problem, filtration through inhomogeneous media. We prove that under suitable restrictions on the coefficients and the nonlinearity exponents the Dirichet problem for equations (1) and (2) admit a. e. bounded weak solutions which belong to the anisotropic analogs of the generalized Sobolev – Orlicz spaces and establish the classes of uniqueness of bounded solutions. Localization properties of weak solutions are discussed for dissipative function $f(x,u)$, i. e., $-f(x,r)r \geq C|u|^{\sigma}$, where C, σ are some positive constants. Using a modification of the method of local energy estimates [1] we show that solutions of equations (1), (2) may identically vanish on a set of nonzero measure either due to a suitable diffusion-absorption balance, or because of strong anisotropy of the diffusion operator. The presentation follows papers [2–4].

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