BOUNDARY VALUE PROBLEMS FOR THE STRING EQUATION, SOME NEW LINKS WITH GEOMETRY, ANALYSIS AND ALGEBRA PROBLEMS

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The communication is devoted to a connection between ill-posed boundary value problems in a bounded semialgebraic domain for partial differential equations and the Poncelet problem, recently revealed by authors. The Poncelet problem is one of famous problems of projective geometry and it by itself has numerous links with a set of different problems of analysis and physics. Investigations of ill-posed boundary value problems in bounded domains for partial differential equations go back to J. Hadamard. The solution uniqueness of the Dirichlet problem for the string equation $u_{xy} = 0$ in Ω, $u|_C = \phi$ on $C = \partial\Omega$ in a bounded domain Ω is connected with properties of John automorphism $T : \partial\Omega \to \partial\Omega$. In particular, there is the following *sufficient condition of uniqueness:* The homogeneous Dirichlet problem has only trivial solution in the space $C^2(\overline{\Omega})$ if the set of periodic points of T on C is finite or denumerable. We consider this problem in a bounded semialgebraic domain, the boundary of which is given by some bi-quadratic algebraic curve $F(x, y) := \sum_{i,k=0}^{2} a_{ik} x^i y^k = 0$. We show the John mapping in this case is the same as Ponselet mapping in some rational parametrizations of conics. From it we obtain

Theorem. *For generic bi-quadratic curve the Dirichlet problem has non-unique solution if and only if corresponding Poncelet problem has periodic trajectory.*

From Poncelet theorem we obtain if there exists some periodical point then each point of C is periodical with the same period. On the other hand a Baxter parametrization allows write the Poncelet mapping by means of elliptic Jacoby functions and obtain a criterion of existence of periodical points and a criterion of uniqueness breakdown for above the Dirichlet problem.

In turn the solution uniqueness of the Dirichlet problem is equivalent to solution uniqueness of some class of boundary value problems for the same equation on C and is equivalent to an indeterminacy of some moment problem on C: $\exists \alpha(s) \neq 0, \forall k = 0, 1, \ldots \int_C [x(s)]^k \alpha(s)\, ds = \int_C [y(s)]^k \alpha(s)\, ds = 0$, where (x, y) are Cartesian coordinates of point on C parametrized by s.

Except for that a Cayley determinant criterion of periodicity of Poncelet problem for case of even period can be understood as a criterion of solvable for algebraic Pell – Abel equation $P^2 - RQ^2 = 1$, where for given polinomial R of the order 4 it is required to find polinomials P, Q. The last problem has connections with a lot of different problems of analysis also.