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UNLOCAL STABLE INVARIANT MANIFOLDS FOR THE GINZBURG – LANDAU EQUATION

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One of possible approaches to solve (local) stabilization problem for nonlinear evolution PDE in a neighborhood of its steady-state solution $\widehat{v}$ is based on using stable invariant manifold defined in this neighborhood of $\widehat{v}$ [1]. In order to extend stabilization theory on the case when initial condition of stabilized equation is far from $\widehat{v}$ one has to be able to construct stable invariant manifold „in whole“, i.e. outside small neighborhood of $\widehat{v}$.

In a bounded domain $\Omega \subset \mathbb{R}^3$ the Ginzburg – Landau equation is considered:

$$\partial_t v(t,x) - \nu\Delta v - v + v^3 = 0, \quad x \in \Omega,\ t > 0 \tag{1}$$

with zero Dirichlet condition on $\partial\Omega$, where parameter $\nu > 0$ is small enough. Then (1) has several steady-state solutions. Let $\widehat{v}$ be one of them. Let $\{e_j, \lambda_j\}, \lambda_1 \leqslant \dots \lambda_N < 0 < \lambda_{N+1} \leqslant \dots \lambda_k \to \infty$ as $k \to \infty$ be eigenfunctions and eigenvalues of linearization on $\widehat{v}$ of space part of (1). We decompose the phase space $V = H^2(\Omega) \cap H_0^1(\Omega)$ of (1) on $V_+ \oplus V_-$ where $V_+ = [e_1, \dots e_N], V_- = [e_{N+1}, \dots]$. The set $M_- \subset V$ contained $\widehat{v}$ is called stable invariant manifold if for each $v_0 \in M_-$ the solution $v(t,\cdot,v_0)$ of (1) with initial condition v_0 belongs to M_- for every $t > 0$, and $\|v(t,\cdot,v_0) - \widehat{v}\|_V \leqslant \exp(-rt)$ as $t \to \infty$ for certain $r > 0$. Moreover

$$M_- = \{\widehat{v} + v_- + F(v_-), v_- \in \mathcal{O}(V_-)\} \tag{2}$$

where $\mathcal{O}(V_-)$ is a neighborhood of the origin in V_-, and $F : \mathcal{O}(V_-) \to V_+$ is a map satisfying $\|F(v_-)\|/\|v_-\| \to 0$ as $\|v_-\| \to 0$.

Existence theorem for invariant manifold M_- is well know when the neighborhood $\mathcal{O}(V_-)$ from definition (2) belongs to a ball $B(V_-, r) = \{v_- \in V_- : \|v_-\|_{V_-} \leqslant r\}$ of small enough radius r. We prove existence of M_- with neighborhood $\mathcal{O}(V_-)$ from (2) which is not contained to the ball $B(V_-, r)$ with arbitrary big r

Introduce the following subspace V_-^k of $V_- : V_-^k = [e_k, e_{k+1}, \dots]$ with $k > N$, and in the space V_- define the following „cruciform“ set: $CR_{r,\rho}(k) = B(V_-, r) \cup B(V_-^k, \rho)$.

Theorem. *For a certain $r > 0$ and for arbitrary $\rho > 0$ one can find $k > N$ such that there exists a neighborhood $\mathcal{O}(V_-)$ satisfying conditions: i) $CR_{r,\rho}(k) \subset \mathcal{O}(V_-)$, ii) There exists a stable invariant manifold (2) with this $\mathcal{O}(V_-)$.*

The proof is based on the theory of analytical stable invariant manifolds. See its local version in [2]

There are examples of ODE when a stable invariant manifold is defined only locally. In the case of parabolic PDE and Navier – Stokes system stable invariant manifolds are always infinite dimensional. That is the reason why they can be extended up to unbounded set.

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