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SOME ASPECTS OF FRACTIONAL DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS

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Our report is devoted to some aspects of the so-called differential equations of fractional order in which an unknown function is contained under the operation of fractional differentiation. A survey of classical and modern results in the theory of ordinary and partial differential equations with fractional derivatives is given.

The existence and uniqueness theorems to Cauchy-type and Cauchy problems for ordinary nonlinear differential equations with Riemann – Liouville, Hadamard and Caputo fractional derivatives are proved by using the method based on the reduction of the considered problems to the equivalent one-dimensional Volterra integral equations.

The operational and compositional methods to solve in closed form some classes of fractional differential equations are presented. Methods of the one-dimensional Laplace, Fourier and Mellin integral transforms are applied to the explicit solutions of some classes of linear ordinary differential equations with Liouville and Caputo fractional derivatives and of the corresponding Cauchy-type and Cauchy problems.

Method of integral transforms is established to construct solutions of Cauchy-type and Cauchy problems for two- and multi-dimensional differential equations with partial Riemann – Liouville and Caputo fractional derivatives with respect to the time generalizing classical diffusion and wave equations, and for fractional evolution equations involving the Liouville partial fractional derivative with respect to the space together with the Riemann – Liouville or Caputo fractional derivative with respect to the time.

Explicit solutions of the considered equations and problems are expressed in terms of the Mittag – Leffler functions and some of their generalizations and modifications, of the Wright generalized hypergeometric functions and of the H-function; for example, see [1, Chapter 1] and [2, Chapters 1 and 2].

Some applications are given. In particular, a new fractional model for the super-diffusion processes involving the partial fractional derivative of a function with respect to another function is presented.

Most of the results were presented in the book [1].

Problems and new trends of research are discussed.

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REFERENCES

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