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# LOCAL STRUCTURE OF THE MORSE SYSTEM OF TWO NON-AUTONOMUS ORDINARY DIFFERENTIAL EQUATIONS

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We consider a smooth system of ordinary differential equations

$$\frac{dx^i}{dt} = f^i(t, x). \tag{1}$$

Here $t \in T \subset R$ is time coordinate, vector $x$ belongs to two-dimensional space of phase coordinates $M \subset R^2$, $f(t, x)$ is a certain smooth vector-function.

Definition 1. We shall say that the point $t_0 = 0, x_0 = 0 \in T \times M$ is the critical point of the system(1) at an instant $t_0$ if $f(t_0, x_0) = 0$ for $i = 1, \ldots, 2$.

We can consider the vector-function $x(t)$ as a result of Cartesian product of maps $x^i : T \longrightarrow X^i$, where $X^i \subset R$, $i = 1, \ldots, 2$.

Note that we can consider also the instant $t = t_0$ as the critical point for each of maps $x^i : T \longrightarrow X^i$.

Definition 2[1]. The critical point $t_0$ is the regular point for the function $x^i(t)$ if

$$\frac{d^2x^i}{dt^2}|_{t_0} \neq 0.$$

Definition 3. We shall say that the system (1) is the Morse system in a neighbourhood of the critical point $t_0, x_0$ if $x^i(t) = \alpha_i t^2$ for each solution $x^i(t)$ of the set (1) ( $\alpha_i = \text{const} \neq 0$ ).

Let the functions $x^i(t)$ in a neighbourhood of the critical point $t_0 = 0, x_0 = 0$ have the form

$$x^i(t) = \alpha_i t^{k_i+1} + O(t^{k_i+2}), \quad (i = 1, 2)$$

where $\alpha_i = \text{const}$, $i = 1, 2$, $k_1 = k$, $k_2 = m$ ( $k, m \geq 1$ ). Then for the given integral manifold we have

$$\frac{dx^i}{dt} = f^i(t, x(t)) \approx \alpha_i (k_i + 1) t^{k_i}. \tag{2}$$

We determine a class of equivalence of smooth functions $f^i(t, x)$, which satisfy to the condition (2).

**Theorem.** *For Morse equations system (1) ( $k_1 = k_2 = 1$ ) the functions $f^i(t, x)$ in a neighbourhood of the critical point $t_0, x_0$ have the form*

$$f^i(t, x) = t(2\alpha_i + t\psi^i_{00}(t, x)) + \sum_j x^j \psi^i_j(t, x), \quad (j = 1, 2)$$

*where $\psi^i_{00}(t, x), \psi^i_j(t, x)$ are smooth functions.*

For example let the considered system contains one equation

$$\frac{dx}{dt} = \beta(x + \beta t).$$

This equation has the solution $x(t) = \exp(\beta t) - 1 - \beta t$ under the condition $x(0) = 0$. In a neighbourhood of the critical point ( $0, 0$ ) this solution has the form $x(t) = (\beta t)^2/2 + O(t^3)$.

## REFERENCES

1. *Poston T., Stewart I. N.* Catastrophe Theory and Its Applications. London: Pitman, 1978.