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ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO HYPERBOLIC SYSTEMS WITH TIME DELAY IN THE BOUNDARY CONDITIONS

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Hyperbolic systems with time delay arise during mathematical modeling of countercurrent chemical reactors with recycle, when some substances return partly to the entrance of the reactor after exiting with some time delay that need for transportation of this substancies (via tubes, mechanically, etc.). It is known that pipe reactors of ideal displacement with the recycle can have several stationary solutions. Therefore the question of their stability arises.

We consider the boundary value problem in the half-strip $\Pi = \{(x,t):\ 0<x<1,\ \ t>0\}$:

$$U_t - L_{\mathcal{A}}U = F(x,t,U), \quad (x,t)\in\Pi, \tag{1}$$

$$\sum_{k=0}^{m}(A_kU(0,t-\tau_k)+B_kU(1,t-\tau_k)) + \sum_{r=0,1}\sum_{k=1}^{m}(\int\limits_0^{\tau_k}\Phi_k^r(\xi)U(r,t-\xi)\,d\xi) = 0, \tag{2}$$

$$U(x,t)|_{\Gamma} = \bar{U}(x,t). \tag{3}$$

Here $U(x,t)$ is an n-dimensional vector of the unknown functions, $F(x,t,U)$ is an n-dimensional vector of smooth functions; $L_{\mathcal{A}}U = -K(x)U_x + A(x)U$, where $K(x)$ is the diagonal matrix with the entries $k_i(x)\neq k_j(x)\neq 0$, $(i\neq j)$, $A(x)$, A_k, B_k, $\Phi_k^r(\xi)$ are $n\times n$ matrices; $n\geq 2$, $0=\tau_0<\tau_1<\ldots<\tau_m$ are fixed reales (the delay times), $m\geq 0$.

In [1] was considered well-posedness of linear problem (1)–(3). In the case when $F(x,t,U)\equiv 0$ and the eigenvalues of the spectral problem corresponding to the system (1), (2) lie on the left half-plane $Re\lambda<-\gamma$ ($\gamma>0$) the estimate

$$\|U(x,t)\|_{C^1[0,1]} \leqslant Ke^{-(\gamma-\varepsilon)t}\|\bar{U}(x,t)\|_{C^1(\Gamma)}, \quad \gamma-\varepsilon>0,$$

for $t>0$ was proved. This estimate makes possible to justify the linearization prinziple for analysis of stability of stationary solutions to the nonlinear problem (1)–(3) if the right part F independent of t evidently.

Let‘s note that the linear hyperbolic systems with time delay in the boundary conditions, as a systems with decomposable boundary conditions without delay [2], have a property of increase of smoothness of solutions by executing some algebraic ratioes between entries of the matrieces $K(x)$, $A(x)$, A_k, B_k, $k=0,\ldots,m$.

Definition. Let‘s speak that the problem (1)–(3) has *the property of increase of smoothness of solutions up to order* k, if there exists such number $T(k)>0$, that any solution $U(x,t)$ of the problem will be k times continuously differentiated for $t>T(k)$. In the case of linear problem $F(x,t,U)\equiv 0$ the solution satisfies for $t>T(k)$ the estimate

$$\|D_{x,t}^{\alpha,\beta}U(x,t)\|_{C[0,1]} \leqslant K(t)\|\bar{U}(x,t)\|_{L_2(\Gamma)},$$

where $\alpha+\beta\leqslant k$, the constant $K(t)$ independent of function $\bar{U}(x,t)$ but depend on the coeffitients of the problem.

For the linear system (1), (2) is isolated the class of named P-regular boundary conditions. The following theorem is proved:

Theorem. *Let* $F(x,t,U) \equiv 0$, $A(x)$, $K(x) \in C^{\mathbf{k+2}}[0,1]$, $0 = \tau_0 < \tau_1 < \ldots < \tau_m$ *— fixed reales. P-regularity of the boundary conditions (2) is a sufficient condition, that the problem (1)–(3) has the property of increase of smoothness of solutions up to order* k *, if* $\bar{U}(x,t) \in C^1(\Gamma)$ *and* $\Phi_l^r(\xi) \in C^1[0,\tau_l]$ *,* $(l = 1,\ldots,m)$*;* $k \geq 0$.

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