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A PRIORY ESTIMATES FOR SOME CLASS OF QUASILINEAR HYPERBOLIC EQUATIONS

© Nurkhat Nurlybayev

nurhat@gmail.com

Centre of Physical and Mathematical Research of National Academy of Sciences, Almaty, Kazakhstan

A priori estimates of solutions to the Cauchy problem for some class of quasi-linear hyperbolic equations are obtained. For these equations the positiveness of solutions and in some special cases the global solvability are proved.

We consider the Cauchy problem for the following class of equations

$$\frac{d}{dl}u(t,\mathbf{x},\mathbf{c})+u(t,\mathbf{x},\mathbf{c})Q(t,\mathbf{x},\mathbf{c},u(t,\mathbf{x},\mathbf{c}))=P(t,\mathbf{x},\mathbf{c},u(t,\mathbf{x},\mathbf{c})), \tag{1}$$

where $\frac{d}{dl}=\frac{\partial}{\partial t}+\mathbf{c}\nabla_{\mathbf{x}}$, $\mathbf{x}$ and $\mathbf{c}$ are n-dimensional vectors.

The following assertion takes place for this equation.

Lemma. *Let* $P(t,\mathbf{x},\mathbf{c},u(t,\mathbf{x},\mathbf{c}))>0$ *if* $u(t,\mathbf{x},\mathbf{c})>0$, $Q(t,\mathbf{x},\mathbf{c},u(t,\mathbf{x},\mathbf{c}))<\infty$ *if* $u<\infty$, $u(0,\mathbf{x},\mathbf{c})=u_0(\mathbf{x},\mathbf{c})>0$. *Then* $u(t,\mathbf{x},\mathbf{c})$ *is positive for all* $t>0$.

Proof. Transforming the Boltzmann equation

$$(\frac{\partial}{\partial t}+\mathbf{c}\frac{\partial}{\partial \mathbf{x}})f(t,\mathbf{x},\mathbf{c})=\int\big[f(t,\mathbf{x},\mathbf{c}')f(t,\mathbf{x},\mathbf{c}_1')-f(t,\mathbf{x},\mathbf{c})f(t,\mathbf{x},\mathbf{c}_1)\big]gd\sigma(g)d\mathbf{c}_1$$

and its discrete model [1]

$$\Big(\frac{\partial}{\partial t}+c^i\nabla_x\Big)u_i(t,x)=\sum_{l\neq j,k\neq i}^{n}\mu_{ij}^{kl}(u_k(t,x)u_l(t,x)-u_i(t,x)u_j(t,x))$$

to the form (1) we get that the solution to Cauchy problem for these equations will be positive if initial data are positive.

Applying this approach to the Carleman discrete model [2] and its generalization

$$\Big(\frac{\partial}{\partial t}+c^i\nabla_x\Big)u_i\equiv\frac{d}{dl_i}u_i=\sum_{l,k\neq i}\mu_{kl}^{ii}(u_ku_l-u_i^2),\qquad \sum_{l,k\neq i}\mu_{kl}^{ii}=1$$

we get the upper estimates for solutions to these equations

$$||u_i(t,x)||_C\leq\max\big\{||u_i^0(x)||_C\big\}.$$

This implies their global solvability.

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