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THE RIEMANN – HILBERT PROBLEM FOR A CLASS OF MODEL VEKUA EQUATIONS WITH SINGULAR DEGENERATION

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In the talk we consider model equations

$$\frac{\partial v}{\partial \overline{z}} - \frac{\lambda + \delta |z|^s}{2\overline{z}} \overline{v} = 0,$$

depending on parameters λ, $\delta \in \mathbb{C} \setminus \{0\}$ and $s \in \mathbb{N}$, with the singular degeneration at $z = 0$ in the region $G \setminus \{0\} = \{z \colon 0 < |z| < 1\}$ and with the boundary value condition

$$\Re v|_\Gamma = f(t).$$

If $\lambda = 0$ this equation is included in the more general class of equations studied by I. N. Vekua [1].

We have proved [6]:

Theorem. *Let $\lambda > 0$, $\delta > 0$, $s \in N$ and $f \in C^{1,\alpha}(\Gamma)$, $0 < \alpha \leqslant 1$. Then the solution v of this problem is uniquely determined and v is given by the explicit formula. This solution belongs to the class*

$$C(\overline{G}) \bigcap C^1(G \setminus \{0\}).$$

Remark 1 (Another known results). This problem is uniquely solvable in the cases

a) if $\delta = 0$, and the solution coincide with the Usmanov' solution [4];

b) if $\lambda \neq 0$, $\arg \lambda \neq \pi$, and δ is small [4];

c) if λ is small, $\arg \lambda \neq \pi$, and δ is arbitrary [5];

d) recently, A. Timofeef and his student from Syktyvkar University have turn up another value of λ and δ for which the conclusion of Theorem is true . This paper will by published soon and these results will be in the talk.

In general case (λ, $\delta \in \mathbb{C} \setminus \{0\}$) we get to explicit formulas of solutions of this equation, depending on parameters $a_0 \in \mathbb{R}$, $c_k \in \mathbb{C}$, $k \in \mathbb{N}$. Finally, we've proved that the solvability of this problem is equivalent to existence zeros for the functions $h_k(\lambda, \delta)$. These functions are given by series.

Remark 2. In general case we don't know the existence of λ, δ, k such that $h_k(\lambda, \delta) = 0$. But from our formulas we can see that

a) The Fredholm alternative take a place, i. e., either the homogeneous problem has a nontrivial solution or the nonhomogeneous problem is solvable for all f;

b) The number n of linear independent solutions of the homogeneous problem is equal to the number n' of the linear independent solvability conditions (or $n = n' < \infty$ or $n = n' = \infty$).

Calculation experiments show that functions $h_k(\lambda, \delta) \neq 0$ if number $k \geq N$.

REFERENCES

1. *Vekua I. N.* Generalized Analytic Functions. Moscow: Nauka, 1988 (in Russian).
2. *Mikhailov L.* A New Class of Singular Integral Equations and Its Applications to Differential Equations with Singular Coefficients. Berlin: Akademie-Verlag, 1970.
3. *Usmanov Z.* About one class of generalized Cauchy – Riemann systems with a singular point // Siberian Math. Journal. 1970. V. 14, N 5. P. 1076–1087 (in Russian).
4. *Usmanov Z.* Generalized Cauchy–Riemann Systems with a Singular Point. Doushanbe: Ac. Sc. Rep. Tadj., 1993. 245 p. (in Russian).
5. *Reissig M., Timofeef A.* Special Vekua equations with singular coefficients // Applicable Analysis. OPA. Malaysia. 1999. V. 73(1-2).
6. *Saks R. S.* Riemann – Hilbert problem for new class of model Vekua equations with singular degeneration // Applicable Analysis. OPA. Malaysia. 1999. V. 73(1-2). P. 201-211.