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MULTIPHASE MODELS WITH MEMORY EFFECT ARISING FROM HOMOGENIZATION

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Multiphase flows equations or multiphase models are usually derived by using the concept of multivelocity continuum and the assumption of interpenetrating motion of the components. In a sense, this concept means that several materials are simultaneously present at each spatial point under consideration. Here we discuss another approach to particular multiphase models that is arisen from homogenization theory.

For this purpose, the homogenization of unsteady diffusion processes in a periodic medium consisting of several materials with widely different properties is considered. The conductivity of one of the materials is assumed to be considerably lower than the conductivity of the other materials. The diffusion processes under study are governed by parabolic equations with coefficients depending on two small positive parameters ε and σ. The microscale parameter ε determines the period of the coefficients in these equations, which corresponds to the assumption that the medium under study has a periodic structure with period ε. The inverse of σ characterizes the scatter of the conductivities in these equations, which corresponds to the assumption that one of the materials has a very low conductivity as compared to the others.

We present homogenized (limit as $\varepsilon \to 0$ and $\sigma \to 0$) equations whose solutions approximate the solutions to the equations under consideration and estimate the accuracy of such approximations. Under certain assumptions on the geometry of the periodic distribution of the constituting materials in space, the homogenized equations form a system of equations coupled through exchange coefficients. The coefficients characterize dynamic diffusion exchange between the materials viewed as components of the medium under consideration and are involved in the homogenized equations via convolution operators with respect to the time variable. Such terms with time convolution in equations has a well-known mechanical interpretation corresponding to the memory effect arising in the homogenized medium.

The homogenized coupled equations and accuracy estimate admit also a natural interpretation of the multiphase flows equations for particular diffusion models. Each equation of such homogenized system characterizes diffusion in the domain occupied by a particular (well conducting) material, and diffusion exchange between these materials is determined by the exchange coefficients coupling the equations. Note also that the equations for the general multiphase models are generally not closed and contain terms that cannot be determined without making additional theoretical assumptions or conducting additional experimental studies, whereas the homogenized multiphase models discussed here for the diffusion problems are given in terms of mathematically defined quantities. Special case of such homogenized models is a well-known model of parallel flows in mechanics of porous media.

Some cases of the homogenized multiphase models were derived and justified in [1–3].

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