Международная конференция "Дифференциальные уравнения, теория функций и приложения", 2007, с. 659–659

УДК 514.763.85

METRIC PROPERTIES OF A MANIFOLD WHICH DEFINED BY THE TWO-POINT CORRELATION TENSOR

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Turbulence is outstanding unsolved multi-scale nonlinear problem of hydrodynamics. Turbulence is convenient name for the phenomenon which is observed in a very large number of flows the properties of which (velocity vector, pressure, vorticity, density, etc ...) undergo random fluctuations created by the presence of numerous eddies of various sizes and, as a result, very extremely irregular in space in time, with broad frequency ranges. Fluctuations of instantaneous flow characteristics (pressure, velocity, vorticity) depend both of space and time; they occur over a very wide range of scales. The smaller scales are settled by the fluid viscosity while the largest are the most often limited by the space available to the flows. For large Reynolds numbers the statistical properties of small scale fluctuations are due to dynamical instability occurring on many scales. These instabilities are responsible for an energy flux from large to small scales, where eventually the energy is dissipated. The Richardson cascade is a simple physical picture which suggests that many statistical properties of small scale fluctuations in turbulence, for large Reynolds numbers, should not depend on the details of forcing and dissipation. Such a supposed independence is known as universality hypothesis an it is one of the major open challenges to be under understood from the Navier – Stokes equations. The Kolmogorov theory implies that the statistical properties of turbulence, at small scales, are universal, if this is the case, the nonlinear term of the Navier-Stokes equations are playing the major role in the determining of such properties. The Navier – Stokes correctly predict how the cascade of turbulent kinetic energy and vorticity accelerates and how the sinews of turbulence stretch themselves into finer and finer scales.

In this talk, the mathematical treatment will be oriented on the metric nature of the so-called scale motions of a turbulent fluid. We primarily deal with isotropic turbulence, emphasis is placed on the study of metric properties and geometric structures of a turbulized domain determined by the two-point correlation tensor $B_{ij}(\vec{x},\vec{x}';t)$ with refereing to Riemannian manifolds and studying the Laplace – Beltrami operator on the so-called crossed product of Riemannian maniflods. In the following we restrict ourselves to isotropic and homogeneous turbulence since only in this case $B_{ij}(\vec{x},\vec{x}';t^*)$ is a symmetric tensor field given on the product $R^3 \times R^3 \simeq R^6$ for a each fixed time t^* (the functional form of this tensor field can be found in any manual on the statistical turbulence) which enables us to introduce into consideration a family of the Riemannian metrics for $t \in R_+$. We give a geometric interpretation of the structural function D_{LL} in the context of studying solutions to a closed model for the von Kármán – Howarth equation in the framework of the theory of geometric evolution equations. This equation is used to investigate the deformation of geometric quantities as this family of metrics evolve under the above-mentioned equation. We show that the behavior of the metrics may still to tell us much about turbulent fluctuations at correlation distances for both small and large scales covering, in particular, the region of the Kolmogorov inertial sub-range. Our aim is to rewrite the closed model for the von Kármán – Howarth equation under consideration in the form of an evolution equation which includes a Laplace – Beltrami type operator defined on a Riemannian manifold. The Laplace – Beltrami operator contains the metric tensor of a Riemannian manifold where this operator is defined. This is a peculiarity of the operator that enables to study an evolution of some geometric and metric quantities which characterizes scaling properties of a turbulent motion. In particular, these investigations gives a mathematical ground to describe the Richardson scenario of the cascade process.

This work was supported by DFG Foundation and by Award 1.8 (2006) for the Integration Projects SB RAS.