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ASYMPTOTIC OF POTENTIALS OF INCLUSIONS IN HIGH-CONTRAST HIGH-FILLED MEDIUM

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Denote $P = [-1,1]^n \subset R^n$ ($n=2$, 3) domain containing a system of not torching domains $\{D_i, i = 1,\dots,N\}$ called inclusions and consider the following boundary value problem

$$\begin{aligned}
&\Delta\varphi = 0 \quad \text{in} \quad Q;\\
&\varphi(\mathbf{x}) = t_i \quad \text{in} \quad D_i, \quad i = 1,\dots,N;\\
&\int_{\partial D_i} \mathbf{vn}\, d\mathbf{x} = 0, \quad i = 1,\dots,N;\\
&\frac{\partial\varphi}{\partial\mathbf{n}}(\mathbf{x}) = 0 \quad \text{on} \quad \partial Q_{lat};\\
&\varphi(\mathbf{x}) = -1 \quad \text{on} \quad \partial D^-, \quad \varphi(\mathbf{x}) = 1 \quad \text{on} \quad \partial D^+;
\end{aligned} \tag{1}$$

∂D^- and ∂D^+ mean the top and bottom boundaries of P, ∂Q_{lat} mean the right and left boundaries of P.

The unknowns in (1) are the function $\varphi(\mathbf{x})$ in domain Q and numbers $\{t_i, i = 1,\dots,N\}$ (potentials of inclusions $\{D_i, i = 1,\dots,N\}$).

The problem goes back to works by Maxwell and Rayleigh. In 1960–2000, the it was investigated asymptotic behavior of the problem under condition that characteristic distance δ between inclusions is small [1]. The modern stage of the analysis of the problem is related to the method of network approximation for high-contrast boundary value problems [2].

The network approximation for the problem (1) is constructed under assumption the inclusion interacts with its neighbors only and flux between i-th and j-th inclusions is equal to $C^{(2)}_{ij}(t_i - t_j)$, where $C^{(2)}_{ij}$ is capacity of these two inclusions R^n. The network approximation has the form

$$\sum_{j\in N_i} C^{(2)}_{ij}(t_i - t_j) = 0, \quad t_i = \pm 1 \quad \text{on} \quad S^{\pm}, \tag{2}$$

where S^+ and S^- are ind ices of quasi inclusions [3, 4] (portions of top and bottom boundaries corresponding to the near-boundary inclusions).

Let us denote solution of the boundary value problem (2) by $\{t^{net}_i, i = 1,\dots,N\}$.

The object of interest was asymptotic of total flux (it is equal to effective conductivity, total energy or capacity of the system of inclusions) as $\delta \to 0$.

The complete proof of asymptotic equivalentness of the total flux corresponding to the problems (1) and (2) as $\delta \to 0$ for disordered (not periodic) inclusions was given not long ago ([3] for a system of planar disks and [4] for general case). It was found that the phenomenon of the network approximation is not an independent problem but a special case of I. E. Tamm asymptotic shielding effect [5, 6].

In [7] (first, to the best knowledge of the author) it was formulated the problem of relationship of potentials $\{t_i, i = 1,\dots,N\}$ and $\{t^{net}_i, i = 1,\dots,N\}$ of inclusions determined from the boundary value problem problem (1) and network problem (2) and it was proved convergence of the potentials of inclusion as $\delta \to 0$ for the the inclusions in the shape of planar circular disks.

In the present paper it is proved that $|t_i - t^{net}_i| \to 0, i = 1,\dots,N$ as $\delta \to 0$ for the inclusions, which shapes satisfy the condition of existence of I. E. Tamm asymptotic shielding effect (in particular, for inclusions with smooth boundaries) [5].

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